

國立成功大學

111學年度碩士班招生考試試題

編 號： 135、139、166

系 所： 航空太空工程學系  
民航研究所  
能源工程國際碩士學位學程

科 目： 工程數學

日 期： 0219

節 次： 第 3 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the solution  $u(x, t)$  of the following initial-boundary values problem

$$(a) \quad \frac{\partial^2 u}{\partial x^2} + \sin 3\pi x = \frac{\partial u}{\partial t} \quad \text{in } 0 \leq x < 1, \quad t \geq 0 \quad (10\%)$$

$$u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x$$

$$(b) \quad \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad \text{in } 0 \leq x < \infty, \quad t \geq 0 \quad (10\%)$$

$$u(x = 0, t) = 0, \quad u(x, t = 0) = a, \text{ where } a \text{ is constant.}$$

2. (a) Use Cauchy's residue theorem to evaluate the given integral along the indicated contour. (10%)

$$\oint_C \frac{\tan(z)}{z} dz, \quad C: |z - 1| = 2.$$

(b) Evaluate the Cauchy principal value of the given improper integral. (10%)

$$\int_0^{\infty} \frac{\cos 3x}{(x^2 + 1)^2} dx$$

3. (20%) Determine the line integral

$$\int_C [2xyz^2 dx + (x^2 z^2 + z \cos yz) dy + (2x^2 yz + y \cos yz) dz]$$

from  $A(0, 0, 1)$  along the  $z$ -axis to  $B(0, 0, 5)$  and finally along the shortest path to  $C(1, \pi/4, 2)$ .

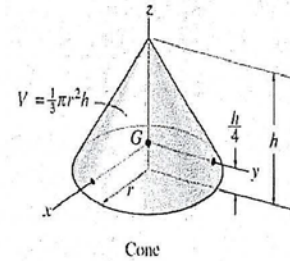
4. (20%) In engineering mechanics, we define the *mass moment*

*of inertia* of a body about an axis of rotation as  $I_A = \int \rho^2 dm$ ,

where  $\rho$  is the distance from  $dm$  to the axis of rotation  $A$ .

Show that  $I_z = \frac{3}{10} mr^2$  for the uniform solid cone (of mass  $m$ )

as shown.



5.

(a) Formulate the *method of least squares* for a linear system of algebraic equations. (10%)

(b) Apply the method of least squares to the *circle-fit* problem, i.e., find the fittest circle to a number of given points in a plane. [Hint: put the circle equation into a *linear-in-parameter* form] (10%)