

# 國立成功大學

## 113學年度碩士班招生考試試題

編 號： 131

系 所： 航空太空工程學系

科 目： 自動控制

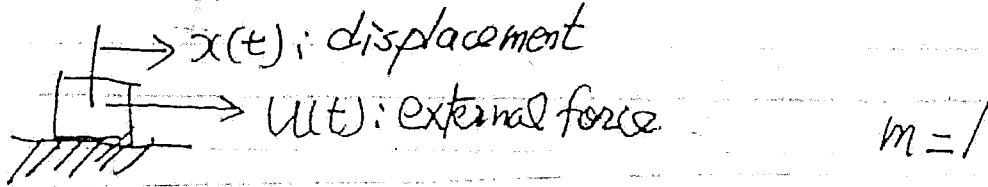
日 期： 0201

節 次： 第 1 節

備 註： 不可使用計算機

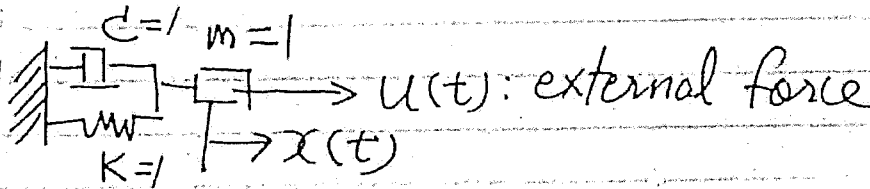
※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。  
 Prob. (一) Derive the state-space representation of the following dynamic systems and their "state transition matrix"

(10%) (i) For system:



Find  $\frac{d}{dt} \bar{x}(t) = A_i \bar{x}(t) + B_i u(t)$   $\begin{matrix} x(t_0) = 0 \\ v(t_0) = 0 \end{matrix}$  and  $e^{(t-t_0)A_i}$  : state transition matrix

(10%) (ii)



Find  $\frac{d}{dt} \bar{x}(t) = \frac{d}{dt} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} = A_{ii} \bar{x}(t) + B_{ii} u(t)$  and  $e^{(t-t_0)A_{ii}}$

(10%) (iii) Discretize system (i) with  $t_k = k \cdot \Delta t, k = 0, 1, 2, \dots$ ,

$u(k) = u(k \cdot \Delta t) = u_k = \text{constant for } k\Delta t \leq t < (k+1)\Delta t$ , so find

$$\begin{bmatrix} x_d(k+1) \\ v_d(k+1) \end{bmatrix} = A_d \begin{bmatrix} x_d(k) \\ v_d(k) \end{bmatrix} + B_k u_d(k), \begin{matrix} x_d(0) = 0 \\ v_d(0) = 0 \end{matrix}$$

(10%) (iv) Find and prove, for  $\begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = A_{\ell s} \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$

$A_{\ell s} \in R^{m \times n}$   $m < n$ , the optimal solution that minimize

$$\frac{1}{2} \sum_{i=1}^n u_i^2$$

(10%) (v) For  $\begin{cases} x_d(4) = 10 \\ v_d(4) = 0 \end{cases}, \Delta t = 1/2$ , with system (i), find  $u_0^*, u_1^*, u_2^*, u_3^*$  that reach the goals and minimize

$$\frac{1}{2} \sum (u_0^2 + u_1^2 + u_2^2 + u_3^2)$$

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(=) Consider the system shown in Fig.3 with  $G(s) = \frac{2}{s^2}$ .

(a). Draw the Bode plot of the system  $G(s) = \frac{2}{s^2}$  (5%)

(b). Design a controller  $C(s)$  such that the resulting system has phase margin of  $45^\circ$  and gain crossover frequency of 10 rad/s. (10%)

(c). Draw the Nyquist plot of the system  $G(s)C(s)$ , and determine the corresponding gain margin by Nyquist plot. (10%)

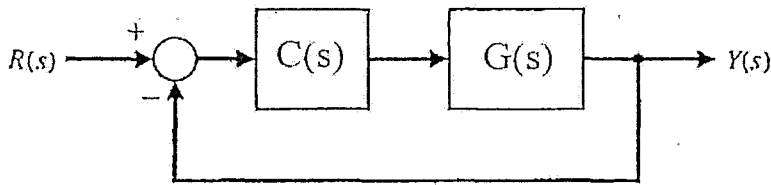


Fig.3

(≡) Consider the system shown in Fig.3 with  $G(s) = \frac{40}{(s+10)(s+2)(s+1)}$  and  $C(s) = k_p + \frac{k_I}{s}$

(a). With  $k_I = 0$ , draw the closed-loop system root locus for  $k_p > 0$ . (10%)

(b). What will be the value of  $k_p$  such that the closed-loop system is critical stable (system output becomes oscillating) and what will be the period of the oscillatory output response (5%)

(c). For  $k_p = 4$ , determine the range of  $k_I$  such that the closed-loop system will be stable. (10%)