國立成功大學

115學年度碩士班招生考試試題

編　號：96

系　所：航空太空工程學系

科　目：動力學

日　期：0203

節　次：第 2 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

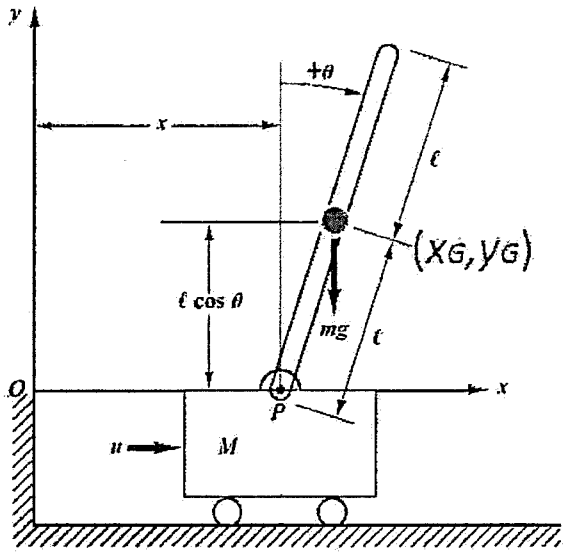
1. (25%) Consider an inverted pendulum system shown as below, where u is the applied force.

(a) Please derive the equations of motions. (10%)

(b) Apply the system linearization with respect to the small angle condition and find the corresponding transfer function from the applied input u to the system output x. (10%).

(c) Examine the stability of the linearized system with respect to the possible equilibrium points. (5%)

PS : Please follow the coordinate system defined in the figure.



2. (25%) Rotation matrices $R(\bullet)$ plays a foundational role in dynamics, particularly in rigid-body kinematics and flight/spacecraft dynamics. Its importance arises from how it encodes orientation as a geometric mapping between reference frames while preserving physical and mathematical structure. Giving yaw (ψ), pitch (θ) and roll (ϕ) angles, which are represented in inertial frame, the corresponding rotation matrices can be expressed as follows:

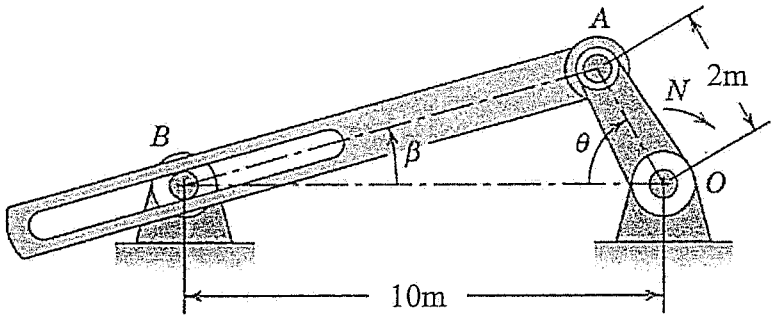
$$R(\psi)=\begin{pmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{pmatrix},\ R(\theta)=\begin{pmatrix}\cos\theta & 0 & \sin\theta\\ 1 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{pmatrix},\ \text{and}\ R(\phi)=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi & \cos\phi\end{pmatrix}$$

(a) Please explain the geometric or physical meaning of the above rotation matrices. Moreover, illustrate one dynamics application example of them. (10%)

(b) Let $R_{321}=R(\psi)R(\theta)R(\phi)$. Please find $R_{321}^{T}R_{321}=?$ (10%)

(c) Let $R_{123}=R(\phi)R(\theta)R(\psi)$. What's the difference between R_{321} and R_{123} (5%)

3. (30%) The angular oscillation of the slotted link is produced by the crank OA, which rotates clockwise at a constant speed of $N = 100$ rev/min. The length of the crank is 2 m, and the fixed pivots O and B are separated by 10 m.
   (a) Determine an expression for the angular velocity $\dot{\beta}$ of the slotted link as a function of the crank angle θ.
   (b) Determine the angular acceleration $\ddot{\beta}$ of the slotted link when $\theta = 45°$. Answers may be given as fractions.



4. (20%) A twin-engine jet (B737-class) travels in a straight line at constant speed. The engines are mounted 6 m from the fuselage centerline. At $t = 0$ s, the right engine produces 30 kN and the left engine produces 20 kN. The aircraft mass is 50,000 kg and its radius of gyration about the vertical axis through the mass center is $k_G = 10$ m. Assuming the aircraft initially has zero yaw rate, determine its yaw angular velocity after 5 s. Answers may be given as fractions.