國立成功大學
115學年度碩士班招生考試試題

編　號：97、117

系　所：航空太空工程學系
能源工程國際碩士學位學程

科　目：工程數學

日　期：0203

節　次：第 3 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. (10%) (a) Find a general solution of the following ODE by the Frobenius method

$$xy'' + (1-2x)y' + (x-1)y = 0$$

(10%) (b) Solve the following system of differential equations:

$$\begin{cases} y_1' = 4y_1 - 8y_2 + 2\cosh t + t\sinh t \\ y_2' = 2y_1 - 6y_2 + 2\sinh t + t\cosh t \end{cases}$$

2. (10%) Solve $y(t) + \int_0^t y(\tau)\sinh(t-\tau)d\tau = 2 - \frac{1}{2}t^2 + (t-2)u(t-2)$

3. (10%) (a) Let $f(x)$ be a 2π −period function defined on $(-\pi, \pi)$ by

$$f(x) = \begin{cases} 0, -\pi < x < 0 \\ x, 0 < x < \pi \end{cases}.$$

Find the Fourier series representation

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

And determine the coefficients a_0, a_n, and b_n.

(10%) (b) Let $a > 0$ and define

$$f(t) = e^{-a|t|}.$$

The Fourier transform is defined by

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt.$$

Compute $F(\omega)$ and write the inverse Fourier transform representation of $f(t)$.

4. (10%) Consider the scalar function $f(x,y,z) = e^x \sin(4y) - z^3$. Find the unit vector in the direction of steepest descent at the point $A(0, \pi, 1)$, that is, the direction with maximum decrease of f at A.

5. (10%) (a) Let C be the triangular path starting from $P(0, 0)$ to $Q(1, -1)$ to $R(1, 1)$ and then returning to P. Calculate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ for the vector field

$$\mathbf{F} = (2x\sin(y^3) - 2y)\mathbf{i} + (3x^2y^2\cos(y^3) + x)\mathbf{j}.$$

(10%) (b) Let S be the surface of the unit cube defined by $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$. Calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{r}$ for the vector field $\mathbf{F} = (3x + y^5z^2)\mathbf{i} + (2y + x^4z^3)\mathbf{j} + (z + x^3y^4)\mathbf{k}$, where $\mathbf{n}$ is the unit outward normal vector on S.

6. (20%) Consider a cylinder of radius a and height H. The base of the cylinder is at $z = 0$ and the top is at $z = H$. Find a function $U(r,z,t)$ which satisfies

$$\frac{\partial U}{\partial t} = k\nabla^2 U$$

in the domain and the stated boundary condition and initial condition.
The boundary condition is that

- $U = 0$ on the surface of the cylinder for all time.

The initial condition is that

- *U within the domain* $= \alpha(r)\beta(z)$ *at time* $t = 0$.

Hint: As it is assumed that U does not depend on θ, the θ ter

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$