國立成功大學 75學年度航空太空工程研究所考試(工程數學試題) 共之頁 第1頁

1). Evaluate

(15%)

$$\int_0^\infty \frac{dx}{x^4+a^4}, \quad a>0.$$

Ans. $\frac{\pi}{2\sqrt{2}\,a^3}$

2). Use the method of variation of parameters to solve the problem

(15%)

$$y''+y=f(x), \quad y'(0)=0, \quad y(\pi)=0$$

where $f(x)\equiv 1$ and $' \equiv \frac{d}{dx}$.

3). Find the derivative of $f=xyz$ at the point $(1,3,2)$ in the direction of the vector $2\hat{i}-\hat{k}$.

(15%)

What is the maximum possible directional derivative of f at that point, and what is its direction?

What is the equation of the tangent plane to the surface $f=$ constant at the point $(1,3,2)$?

4). For the ordinary differential equation $\ddot{x}+2\dot{x}+x=\dot{u}+u$, find the complete solution of $x(t)$ for an input $u(t)=1+t-e^{-t}$ subject to the initial conditions of $x(0)=0$ and $\dot{x}(0)=0$.

(15%)

5). Transform the multiple integral

(15%)

$$I=\iint_A |\sqrt{x+y}|\,dx\,dy$$

to one in terms of ξ and η by means of the relations

$$x=\xi^2+\eta^2, \quad y=2\xi\eta.$$

What is the restriction on the transformation?

Do not attempt to perform the integration.

6). Use the Fourier transform to solve the partial differential equation:

(10%)

$$\frac{\partial u}{\partial t}=\frac{\partial u}{\partial x}, \quad -\infty<x<\infty, \quad 0\le t<\infty$$

with $u(x,0)=f(x)$.

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7). Are the following problems solvable in the domain $0 \le x < \infty$, $0 \le t < \infty$?
(15%) Explain why.

i). $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}$

ii). $\frac{\partial u}{\partial t} = -\frac{\partial u}{\partial x}$

with the boundary conditions

$$u(x,0) = f(x)$$

and

$$u(0,t) \text{ not specified.}$$

Discuss your solutions on the t-x plane.