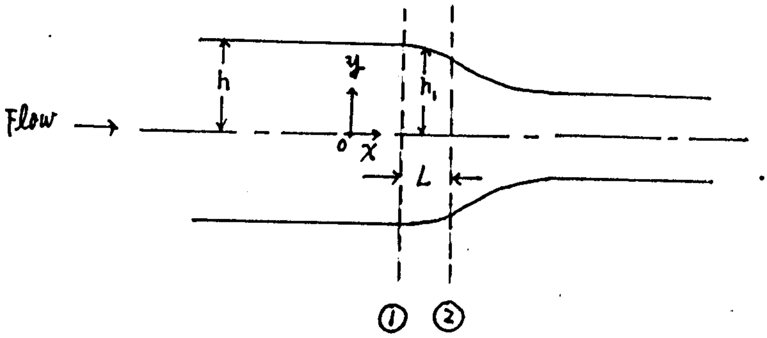
1. Air flows through a 2-dimensional, symmetric contraction channel at low speed (Mach no. ≈ 0)



$h = 5\ m$

$h_1(x) = h\left(1 - \left(\frac{x-1}{L}\right)^2 0.2\right)$

$L = 1\ m$

$x = 1\ m$ at section ①

$x = 2\ m$ at section ②

Assume that the flow is laminar, and the displacement thickness, $\delta^*(x)$, and momentum thickness, $\theta(x)$, between the sections ① and ② can be described as follows:

$$\delta^*(x) = 0.0025\, h\,(x/L)^{-1}$$

$$\theta(x) = 0.001\, h\,(x/L)^{-1}$$

Hence the shape factor H is $\delta^*/\theta = 2.5$, which is a constant.

10% (a). Find the pressure gradient, $\frac{dp}{dx}$, between the sections ① and ② in terms of ρ, U_1, x, L, h, where ρ is the density of air and U_1 is known, the freestream velocity at section ①. Is $\frac{dp}{dx}$ greater than zero throughout this region?

10% (b). The momentum integral equation is given as follows:

$$\frac{d\theta}{dx} + (H+2)\frac{\theta}{U}\frac{dU}{dx} = \frac{\tau_w}{\rho U^2} = C_f$$

where C_f is called the skin friction coefficient. Find C_f at section ②, $x = 2\ m$.

2\. (a) Explain each term in the Reynold transport theorem

7%

$$\frac{D}{Dt}\iiint_{c.\forall} \rho\alpha\, d\forall = \frac{\partial}{\partial t}\iiint_{c.\forall} \rho\alpha\, d\forall + \oiint_{c.s} \rho\alpha\, \vec{V}\cdot\vec{n}\, dA$$

where ρ is the density, α the extensive physical property per unit mass, $c.\forall$ the control volume, $c.s$ the enclosed control surface, $\vec{n}$ the outward normal of surface, $\vec{V}$ the velocity.

(b). suppose the control volume is rigid but is not fixed with respect to an inertia frame, discuss the validity of the Reynold transport theorem.

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(c). Assume no gravitational force. Do you think that the ball in the air, shown in the figure, will turn to left or right? explain it.

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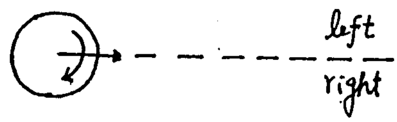
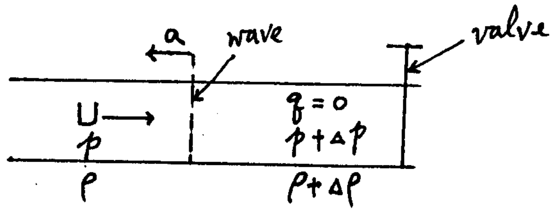


3\. A fluid flowing at speed U in a rigid pipe is stopped by sudden closure of a valve, and a pressure wave propagates up the pipe with speed a as shown in the figure. After the wave, the pressure and density of the fluid are suddenly increased and the velocity $q=0$. If one follows the wave, then the unsteady flow stated above can be reduced to a steady flow. Draw a figure to indicate the velocities before and after the wave in this steady flow and express Δp and $\Delta p/\Delta\rho$ in terms of ρ, U and a.

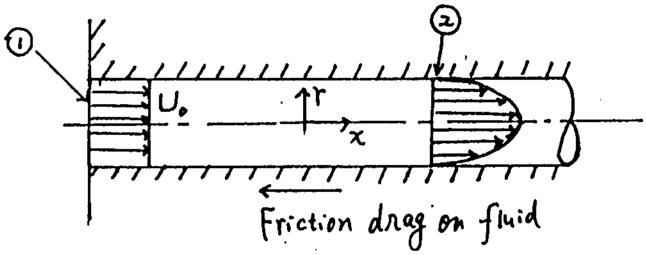
20%



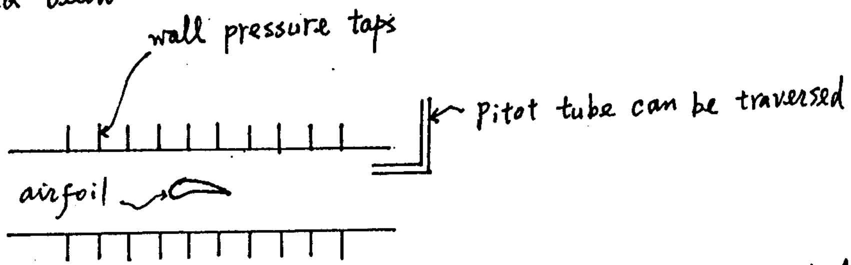
4. 20% The steady incompressible entrance flow shown in a pipe of radius R develops from uniform flow U_0 at section 1 to the laminar paraboloid

$$u = u_{max}\left(1 - r^2/R^2\right)$$

at section 2. Find the wall drag F as a function of p_1, p_2, U_0 and R



5. A wind tunnel arrangement for determining the lift and drag on an airfoil is sketched below



a). 10% Determine an equation and a procedure for obtaining the airfoil lift

b). 10% Due to the boundary layer displacement effect, the main stream velocity will increase. If U is the downstream velocity, and $U(1-\varepsilon)$ the upstream velocity outside of the boundary layer, show that the drag

$$D = \rho U^2\theta\left[1 + O(\varepsilon)\right]$$

where $U^2\theta = \int u(U-u)\,dy$; $H = \dfrac{\delta^*}{\theta}$ and $U\delta^* = \int (U-u)\,dy$. The integration is performed at the downstream section and the limits of integration are large enough so that at the upper limits $U - u \simeq 0$

Note: pressure taps are for the measurement of static pressure, and pitot tube for the dynamic pressure