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1. (a) Describe the state of stress at a point of a continuum. (5%)

   (b) Must the stress tensor be symmetric ? Why ? (5%)

   (c) Let the plane stress components σ_{xx}, σ_{yy}, σ_{xy} and $\sigma_{x'x'}$, $\sigma_{y'y'}$, $\sigma_{x'y'}$ be referred to the right-hand rectangular Cartesian coordinates x, y and x', y', respectively. Show that

$$\sigma_{x'x'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta + \sigma_{xy}\sin 2\theta$$

$$\sigma_{y'y'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} - \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta - \sigma_{xy}\sin 2\theta$$

$$\sigma_{x'y'} = -\frac{\sigma_{xx}-\sigma_{yy}}{2}\sin 2\theta + \sigma_{xy}\cos 2\theta$$

   where θ is the angle of counterclockwise rotation from the coordinate x, y to the coordinate x', y'. (10%)

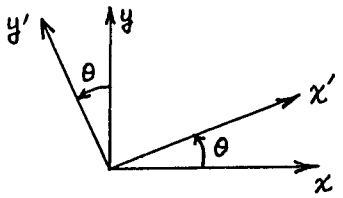

Fig. 1

2. Let E, G and ν be the Young's modulus, shear modulus, and Poisson's ratio of a continuous body, respectively.

   (a) Write the six linear-elastic stress-strain relations. (5%)

   (b) What is the relation among E, G and ν ? Can you prove it ? (15%)

3. Describe (a) Mohr's circle (4%)

   (b) Bernoulli-Euler beam (4%)

   (c) Shear center (4%)

   (d) Strain energy (4%)

   (e) Elastic instability (4%)

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4. A circular cantilever beam is shown in Fig. 2 withstanding a 5000 lb axial force and a 100 lb force acting on attached member AB. Does the plastic deformation in the beam take place ? The beam is made of a ductile metal with a Young's modulus of 30×10^6 psi, a Poisson's ratio of 0.3, and an initial yield stress of 30×10^3 psi. (20%)

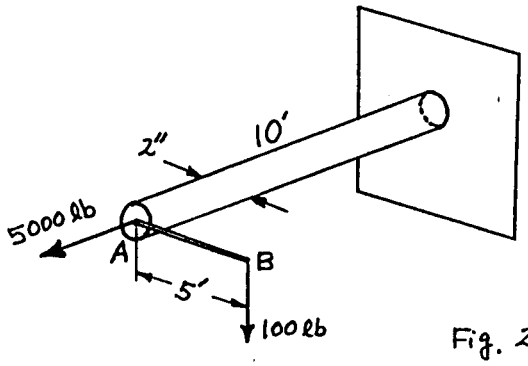

Fig. 2

5. Before the load P was applied to the beam as shown in Fig. 3, the beam was straight and the spring was unstretched. Find the deflection under the load P in terms of EI, L, and the spring constant k. (EI and k are constants.) (20%)

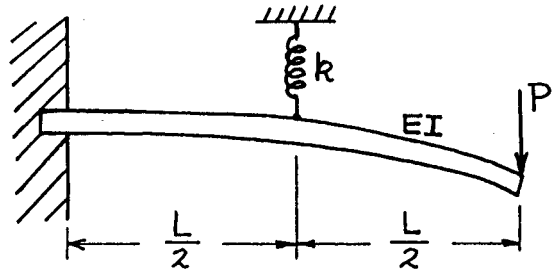

Fig. 3