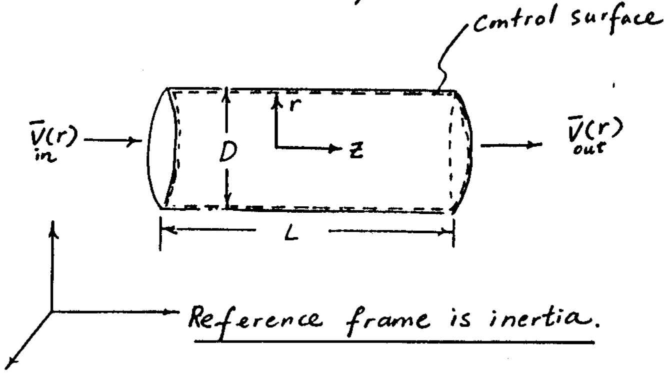
1. (20%) (a) State the Reynolds transport theorem. (2%)

(b) For the following control volume, by Reynolds transport theorem, write

$$\frac{dM}{dt} = \frac{d}{dt}\iiint_{system} \rho \, dV = ?$$

$$\frac{dF_x}{dt} = \frac{d}{dt}\iiint_{system} \bar{V} \, dm = ? \qquad \text{where} \quad dm = \rho \, dV$$

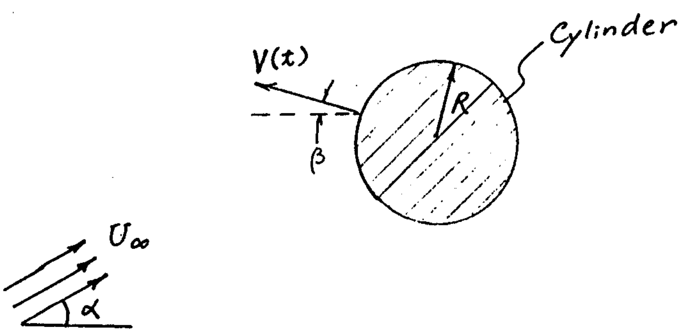


(8%)

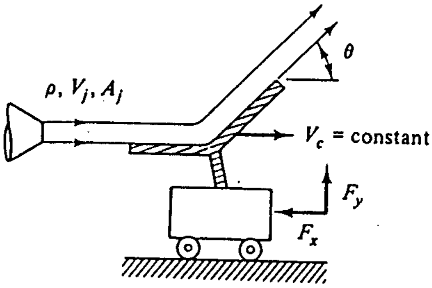
(c) Write the continuity equation for the above control volume. (4%)

(d) Suppose the reference frame is set in motion with constant acceleration, is the above formula you give in part (b) still valid? why? (6%)

2. (20%) A cylinder of radius R is advancing in a parallel airstream illustrated below. Assume that the flow is inviscid and nicely attached on the surface of the cylinder. Find the expression for the velocity on the cylinder surface.



3. (20%) Make an analysis of the case shown below, where the vane moves to the right at constant velocity V_c on a cart. The jet velocity and area are V_j and A_j. The pressure is uniform. Compare the forces F_x and F_y required to restrain the cart and keep it in this non-accelerating condition. Neglect cart weight and friction.

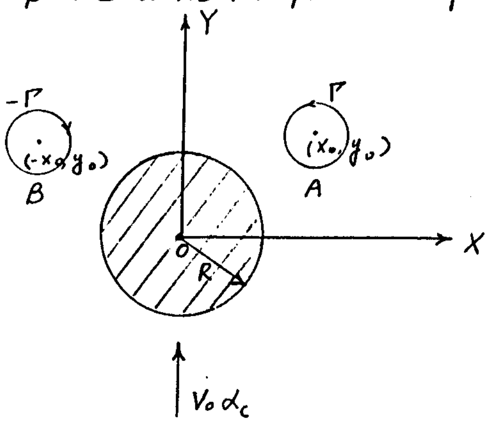


4. (20%) A pair of free vortices are located symmetrically near a circular cylinder of radius R, with same strength and opposite sense as shown. If the freestream velocity is $V_0 \alpha_c$ in Y-axis direction,

(a) find the velocity distribution in the X-Y plane,

(b) find the velocity of free vortex A.

(c) find the locus of Föppl points (i.e. the locus of which the velocity of the pair of free vortices are zero) and plot it on the X-Y plane with the presence of the cylinder.



5. (a) Derive the following equation by using the boundary layer equation
(20%)

$$\nu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\partial(U\delta_1)}{\partial t} + \frac{\partial U}{\partial x}U\delta_1 + \frac{\partial(U^2\theta)}{\partial x} \qquad (1)$$

where δ_1 : displacement thickness $\frac{1}{U}\int_0^\infty (U-u)\,dy$.

θ : momentum thickness $\frac{1}{U^2}\int_0^\infty u(U-u)\,dy$

ν : $\frac{\mu}{\rho}$

U : velocity of potential flow outside the boundary layer

(b) Find out the boundary layer thickness $\delta(x)$ by using above eq. (1) and the velocity distribution

$$\frac{u}{U} = \frac{y}{\delta} \qquad (0 < y \le \delta)$$

$$\frac{u}{U} = 1 \qquad (y > \delta)$$

for the steady, flat plate boundary layer case.