

1. (a) Describe the basic requirements for $\delta = \frac{PL}{EA}$ to be valid.
- (b) (i) Describe the difference between $\tau = \frac{Tr}{I_p}$ and $\tau = \frac{T}{\Delta A_m r}$.
- (ii) Under what conditions both of the torsion formulas can be applied?
- (c) How to modify the flexure formula $\sigma = \frac{My}{I}$ for composite beam and unsymmetric bending problems?
- (d) Briefly explain the Maxwell's reciprocal theorem. Also state the basic requirements for this theorem to be valid.
- (e) Which of the followings are valid for nonlinear inelastic materials. Write down your answer and explain briefly.
- (i) $\epsilon_x = -\kappa y$, (ii) $\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$
- (iii) $\tau = \frac{Tr}{I_p}$, (iv) The principle of virtual work

2. For the two cantilever beams AB and CD, determine the deflection at end B caused by the load P. (see Figure 1)

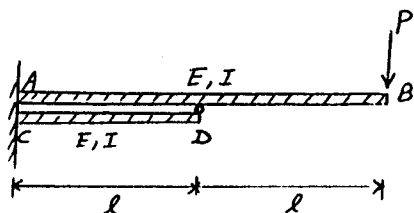


Figure 1

3. A linearly elastic simply supported beam with varying cross section as shown. 15%

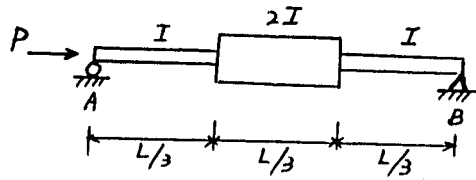


Figure 2

Assuming the beam is homogeneous with Young's modulus E , use the Rayleigh-Ritz method to determine the critical value of P that produces buckling of the pinned-end column.

Note: Rayleigh-Ritz method.

- (i) assume an appropriate mode shape, for example, $\delta \sin \frac{\pi x}{L}$, satisfying geometric boundary condition, i.e., deflection is zero at A and B.
- (ii) calculate the total potential energy PE, i.e.

$$PE = (\text{strain energy of the structure}) + (\text{potential energy of } P)$$
- (iii) minimize total potential energy and obtain P_{cr} .

4. A thin walled z-section is shown in Figure 3. Determine the shear flow distribution due to a shear load S_y applied through the shear center of the section.

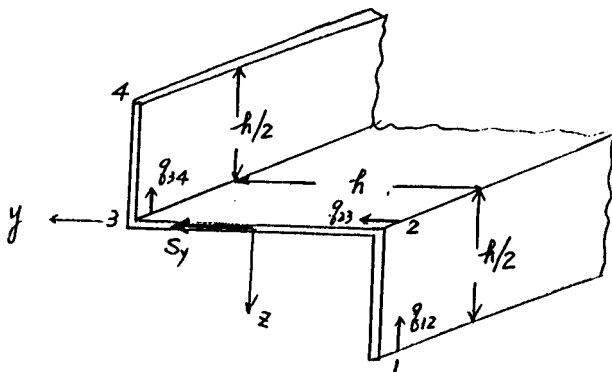


Figure 3

Thickness = t for all webs

$$I_{zz} = \frac{h^3 t}{3}, \quad I_{yy} = \frac{h^3 t}{12}, \quad I_{yz} = \frac{h^3 t}{8}$$

Note that the details of derivation for I_{zz} , I_{yy} , I_{yz} have to be shown. Shear flow notations are q_{12} , q_{23} , q_{34} .

5. A cantilever beam is loaded by its own weight q (assumed uniformly distributed along the beam) and an inclined force P as shown in Figure 4.
- (a) Determine the stresses at points A and B in the figure.
- (b) If the allowable tensile stress and shear stress are limited to σ_0 and τ_0 , respectively. Also, $L \gg d \tan \alpha$, $p \cos \alpha \gg qL$, $30^\circ < \alpha < 60^\circ$. What is the largest permissible value of the load P ?

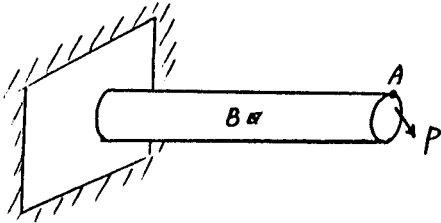
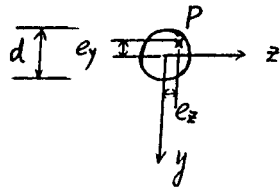
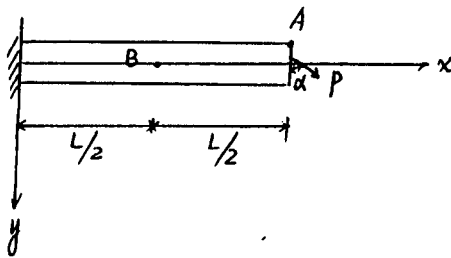


Figure 4

Side-view:



Note: * P is an eccentric inclined force parallel to x - y plane.

* Both A and B are located on the outer surface of the circular cross section.

* Express your solutions in terms of P , q , L , e_y , e_z , d , α , σ_0 , τ_0 .