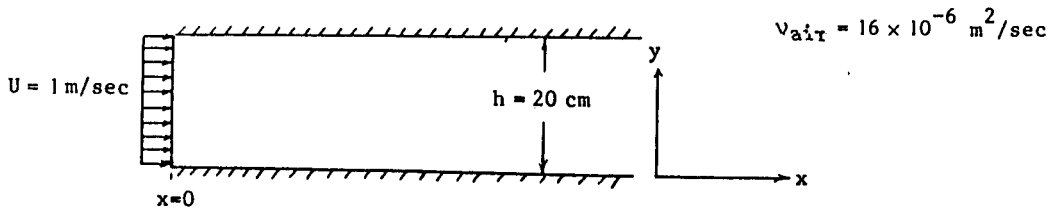


- 20% 1. Air convects through a two-dimensional channel, see the figure. Assume that flow is steady and incompressible, and is uniform in velocity distribution at  $x=0$  (inlet cross section)

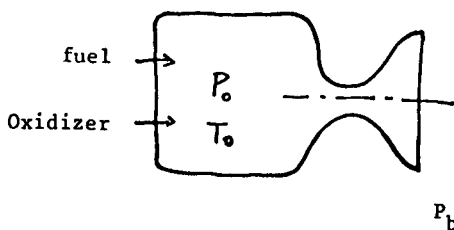


Assume that boundary layer developed on the wall from  $x=0$  to  $1\text{ m}$  is laminar, where the boundary layer thickness,  $\delta$ , the displacement thickness,  $\delta^*$ , and the momentum thickness,  $\theta$ , are expressed as follows.

$$\delta = 5.0 \sqrt{\frac{\nu x}{U}}, \quad \delta^* = 1.7 \sqrt{\frac{\nu x}{U}}, \quad \theta = 0.66 \sqrt{\frac{\nu x}{U}}$$

- 5% (a) One observes a fluid particle convecting from  $x=0$  to  $1\text{ m}$ . At  $x=1\text{ m}$ , this fluid particle is at the edge of the boundary layer developed along the bottom wall. Where would this particle be when initially it passed the cross section  $x=0$ ?
- 10% (b) Denote the velocity of this particle as  $\vec{V}$ . Is  $\frac{D\vec{V}}{Dt} = 0$ ?  $\frac{D}{Dt}$  is the substantial derivative. Give your reasons to support your answer. If your answer is  $\frac{D\vec{V}}{Dt} \neq 0$ , give your estimation of  $\frac{D\vec{V}}{Dt}$  at  $x=1\text{ m}$ .
- 5% (c) Let the streamfunction value along  $y=0$  (bottom wall) be zero. Find the streamfunction value of the streamline which coincides with the path of the fluid particle.

- 20% 2. A rocket motor as shown below



$P_0$ : chamber total pressure  
 $T_0$ : chamber total temperature  
 $P_b$ : back pressure

- 10% (a) Plot the pressure distribution on the nozzle wall and chamber wall.  
 10% (b) With the pressure integration, describe how it generates thrust.

- 20% 3.

- 5% (a) It is known that the complex potential for a flow is

$$w = V_\infty \left( z + \frac{r_0^2}{z} \right) + i \frac{\Gamma}{2\pi} \ln z$$

Find its lift force per unit depth.

10% (b) For a Joukowski airfoil at an angle of attack, we know that it has lift force. On the other hand, since every aircraft has to be accelerated from zero velocity, no vortex exists initially. Use the Kelvin theorem to explain the reason that lift force does exist whenever  $V_\infty > 0$ .

5% (c) Is the result in part (b) valid for the case of viscous flow? Why?

20% 4. Consider a steady, two-dimensional flow with constant density and constant properties passing over a semi-infinite long flat plate at zero incidence. Answer the following two questions and state your reasons as clear as possible.

10% (a) Determine the simplified governing equations (P.d.e. is O.K.) of the Navier-Stokes equations according to the Prandtl's approximation.

10% (b) Assuming the temperature of the free stream is cooler than the wall temperature, what are the comparisons of the momentum boundary layer thickness and the thermal boundary layer thickness as the Prandtl number much greater than one and much less than one, respectively.

20% 5. An endless belt passes upward through a chemical bath with speed  $V$  and picks up a film of liquid of thickness  $h$ , density  $\rho$ , and viscosity  $\mu$ . Gravity tends to drain the liquid down, but the movement of the belt keeps it from completely running off.

12% (a) Determine the velocity distribution across the film.

8% (b) Find the rate of flow at which the liquid is being dragged up by the belt.