1. Consider the second order system as shown in Figure 1. This system has a rate sensor which saturates at 10 volt/sec. The system is to be excited by a sinusoidal input of the following form.

$$r(t) = A(\omega)\sin\omega t$$

Find the bound of the amplitude A, as a function of frequency, ω, so that the rate sensor won't saturate. (Assume that r(t) is started from $t=-\infty$.) (20%)

2. Consider the closed loop system as shown in Figure 2. Assume that G(s) is a proper stable transfer function, and $G_c(s)$ is the closed loop transfer function.

   (a) Show that $G_c(s)$ is stable when $\max_{\omega} |k|\,|G(j\omega)| < 1$. (15 %)

   (b) Let $G(s) = \frac{s-1}{s+1}\frac{s-2}{s+2}\frac{s-3}{s+3}\frac{s-4}{s+4}\frac{s-5}{s+5}$. Find all k such that $G_c(s)$ is stable. (15%)

3. In the system shown in Figure 3.1, mass m=9 kg is subjected to force F(t) acting vertically and undergoing from 0 to 1.0 N at time t=0 (step input). The mass, suspended on a spring of constant k=4.0 N/m, is moving inside an enclosure with a coefficient of friction between the surfaces b=4.0 N-sec/m.

   (a) Using force F(t) as the input variable and position of mass x(t) as the output variable, sketch an approximate step response of the system and determine (1) the system transfer function G(s), (2) the damping ratio ζ, (3) the natural frequency ω_n and (4) the percent maximum overshoot M_p of this system. (15%)

   (b) In order to make the system critically damped, a damper is added as shown in Figure 3.2. Determine the coefficient of the damper b_{ad}. (5%)

4. An airplane with an autopilot in the longitudinal mode has a simplified open-loop transfer function

$$G(s)K(s) = \frac{k(s+1)}{s(s-1)(s^2+4s+16)}$$

Sketch the root-locus plot and determine the range of gain k for stability. (15%)

5. A specific closed-loop control system as shown in Figure 5 is to be designed for an underdamped response to a step input. The specification for the system are

20% > percent overshoot > 10%

settling time < 0.2 sec.

   (a) Identify the desired area for the dominant roots of the closed-loop system.

   (b) Determine the smallest value of r_3, if the complex conjugate roots are to present the dominant response.

   (c) With r_3 obtained in (b), determine the forward transfer function G(s) when the setling time is 0.2 sec and the percent overshoot is 10%. (15%)

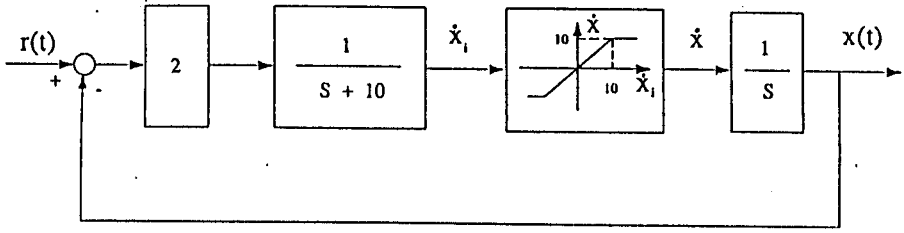


Figure 1

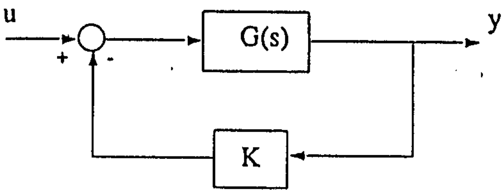


Figure 2

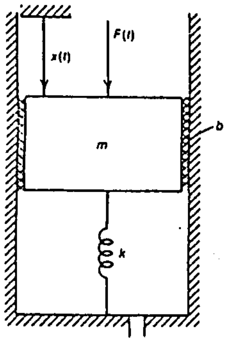


Figure 3.1

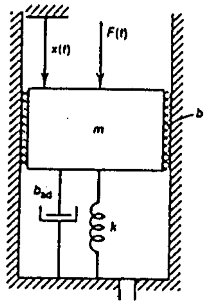
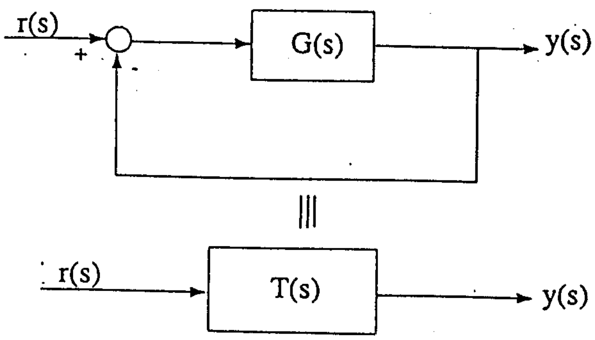


Figure 3.2



$$T(s) = \frac{r_3\omega_n^2}{(s+r_3)(s^2+2\zeta\omega_n s+\omega_n^2)}$$

Figure 5