1. (25%) A propped cantilever beam, fixed at the left end A and simply supported at the right end B, has temperature T_1 on its upper surface and T_2 on its lower surface (see Figure 1a).

(a) Find the reactions for the beam if the beam is made of homogeneous materials with flexural rigidity EI and thermal expansion coefficient α. (see Figure 1b)

(b) Find the reactions for the beam if the beam is made of bimetallic composites with flexural rigidity E_1I_1 and E_2I_2 and thermal expansion coefficients $\alpha_1 = \alpha_2 = \alpha$. (see Figure 1c)

(c) Calculate the absolute maximum stress in the beam for both of problems (a) and (b).

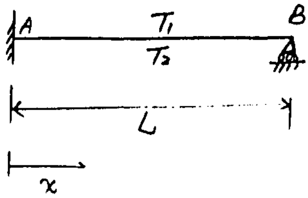


Figure 1a

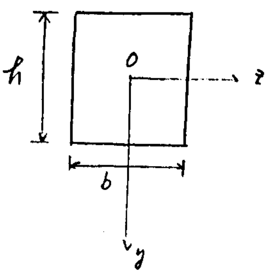


Figure 1b

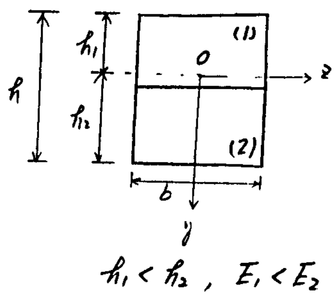


Figure 1c

y, z : neutral axes

E : modulus of elasticity

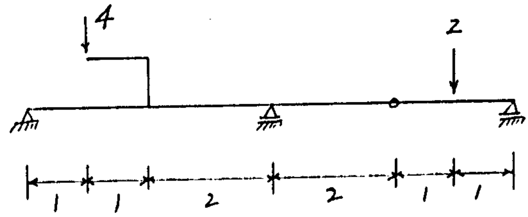
I : moment of inertia about the neutral axis

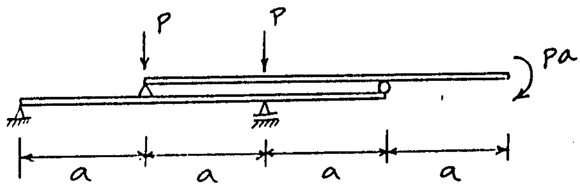
y, z : neutral axes

E_1, E_2 : moduli of elasticity for materials (1) and (2), respectively

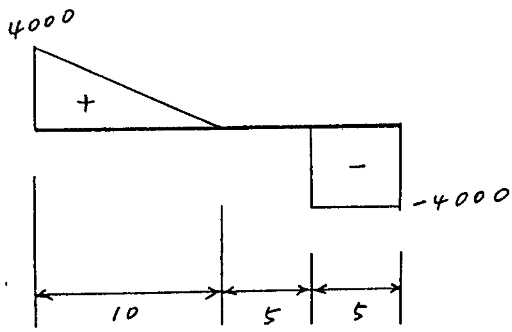
I_1, I_2 : Moments of inertia of cross-sectional area (1) and (2), respectively, about the neutral axis

(25%) 2. (a) Draw the shear-force and bending-moment diagrams for the beams

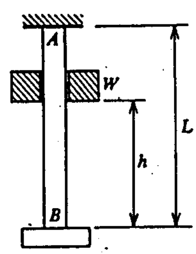




(b) The shear-force diagram for a simple beam is shown below. Determine two possible loading cases on the beam and draw the bending-moment diagrams.



(25%) 3. (a) A weight W, initially at rest, falls from a height h onto a flange at the lower end B of a bar of length L. Find the maximum elongation of the bar and the maximum tensile stress due to the impact. Neglect energy losses during the impact.

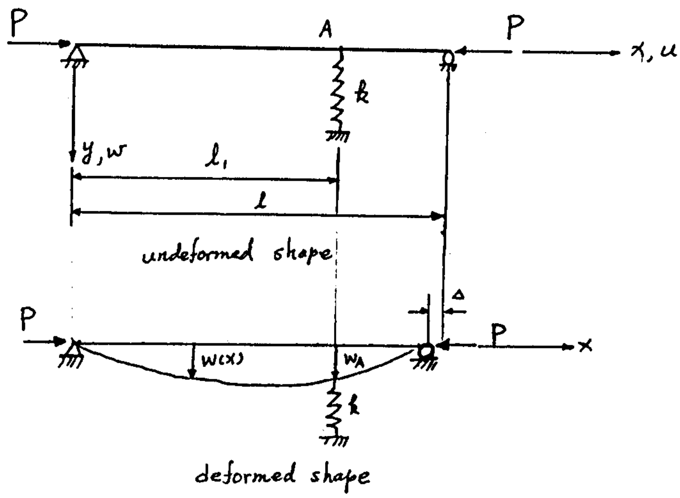


(b) Determine the strain energy in a vertical, prismatic bar hanging under its own weight, if L = length of bar, A = cross-sectional area, E = modulus of elasticity, and γ = weight of bar per unit volume.

(25%) 4. A beam-column, simply supported and constrained at point A with a linear spring, is subjected to a compressive load P. The beam-column has a length ℓ and flexural rigidity EI while the spring has an elastic constant k.

(a) Write the total potential energy for the beam-column.

(b) Use Rayleigh-Ritz method to determine the buckling load P_{cr} for the beam-column.



Hint : $\varepsilon_x = \frac{du}{dx} = -\frac{1}{2}\left(\frac{dw}{dx}\right)^2$

$$\Delta = \int_0^{\ell} \varepsilon_x \, dx$$