(20%) 1. Consider an NACA0012 airfoil with a chord of 1.0 meter and span of 4 meters in an airstream at standard sea level conditions. The freestrem velocity is 80 m/s. The total lift force is 5000 Newtons. By assuming the inviscid flow, answer the following questions. (1), calculate the necessary angle of attack ($\alpha = ?$) based on the two-dimensional thin airfoil theory, such that the lift force per unit span (L) can be approximately evaluated by (6%)

$$c_\ell = \frac{L}{q_\infty S} \sim 2\pi\alpha, \; q_\infty = \frac{\rho_\infty U_\infty^2}{2}, \; S \sim \text{chord} \times 1 \text{ meter}$$

(2), if consider the three-dimensional effect, do you think that the necessary α will be increased or not? Please explain the reason. (4%)

(3), now the NACA0012 airfoil is replaced by a circular cylinder with diameter of 1 meter and the same span of 4 meters. Determine the angular velocity (in rpm) of the circular cylinder in order to provide the same lift force of part (1). (10%)

(20%) 2. A normal shock moves at a constant velocity of 500 m/s into still air (100 kPa, 0° C). Determine the static and stagnation conditions present in the air after passage of the wave, as well as the gas velocity behind the wave.

Table B.1 Normal shock tables ($\gamma = 1.4$)

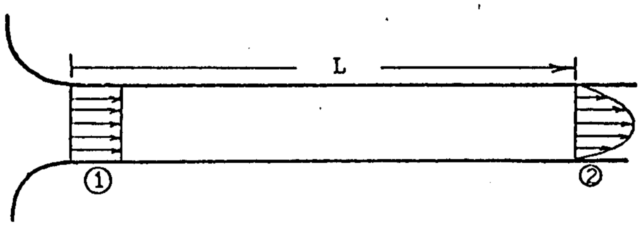
M_1	M_2	p_2/p_1	ρ_2/ρ_1	T_2/T_1	p_{t2}/p_{t1}	p_1/p_{t2}
1.00	1.000	1.000	1.000	1.000	1.000	0.5283
1.01	.9901	1.023	1.017	1.007	1.000	.5221
1.02	.9805	1.047	1.033	1.013	1.000	.5160
1.03	.9712	1.071	1.050	1.020	1.000	.5100
1.04	.9620	1.095	1.067	1.026	.9999	.5039
⋮	⋮					
1.40	.7397	2.120	1.690	1.255	.9582	.3280
1.41	.7355	2.153	1.707	1.261	.9557	.3242
1.42	.7314	2.186	1.724	1.268	.9531	.3205
1.43	.7274	2.219	1.742	1.274	.9504	.3169
1.44	.7235	2.253	1.759	1.281	.9476	.3133
1.45	.7196	2.286	1.776	1.287	.9448	.3098
1.46	.7157	2.320	1.793	1.294	.9420	.3063
1.47	.7120	2.354	1.811	1.300	.9390	.3029
1.48	.7083	2.389	1.828	1.307	.9360	.2996
1.49	.7047	2.423	1.845	1.314	.9329	.2962
1.50	.7011	2.458	1.862	1.320	.9298	.2930
1.51	.6976	2.493	1.879	1.327	.9266	.2898
1.52	.6941	2.529	1.896	1.334	.9233	.2866
1.53	.6907	2.564	1.913	1.340	.9200	.2835
1.54	.6874	2.600	1.930	1.347	.9166	.2804
1.55	.6841	2.636	1.947	1.354	.9132	.2773
1.56	.6809	2.673	1.964	1.361	.9097	.2744
1.57	.6777	2.709	1.981	1.367	.9061	.2714
1.58	.6746	2.746	1.998	1.374	.9026	.2685
1.59	.6715	2.783	2.015	1.381	.8989	.2656

(20%) 3. An incompressible fluid flows in a pipe of radius, R. At the inlet, section 1, the velocity is uniform over the cross-section, with a value V. At section 2, where the flow is laminar and fully developed, the velocity varies with radius according to the relation

$$V = V_{max}\left(1 - \frac{r^2}{R^2}\right)$$

a) Demonstrate that $\frac{V_1}{V_{max}} = \frac{1}{2}$

b) If $\bar{\tau}_w$ is the average wall shearing stress retarding the flow between sections 1 and 2, find the pressure drop $(p_1 - p_2)$ in terms of $V_1, \rho, L, R, \bar{\tau}_w$



(20%) 4. A circular disc of radius a is parallel to and at distance h from a rigid plane, and the space between them is filled with fluid. The disc approaches the plane at a constant velocity U. Assuming that $h << a$, $\frac{\rho h U}{\mu} << 1$, the flow is steady, axisymmetric and dominantly radial. The pressure at the edge of the disc is atmosphere.

(1) Based on the above description and the following simplified Navier-Stokes equations in cylindrical coordinates (see Reference), find out the equations of motion and the boundary conditions which are suitable for the present problem.

(2) From the equations of motion, continuity equation and boundary conditions, determine the pressure distribution on the moving disc.

Reference:

Simplified Navier-Stokes equations in cylindrical coordinates

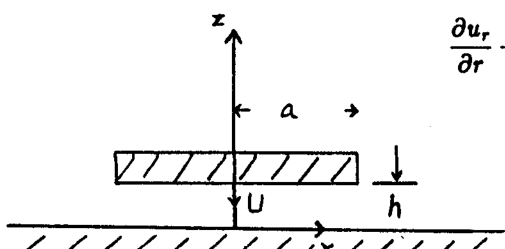
$$\mu\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right) = \frac{\partial p}{\partial r}$$

$$\mu\left(\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2}\right) = \frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$\mu\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right) = \frac{\partial p}{\partial z}$$

Continuity equation

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$



(20%) 5. Steady, incompressible boundary layers develop on the two-dimensional duct walls.

Given: boundary layer thickness $\theta = 6.0\sqrt{\frac{\nu x}{U_0}}$

displacement thickness $\delta^* = 2.0\sqrt{\frac{\nu x}{U_0}}$

momentum thickness $\theta = 0.7\sqrt{\frac{\nu x}{U_0}}$

where U_0 is the velocity of the uniform flow at the inlet of the duct; p_0 is the static pressure of the flow at the inlet of teh duct, and ν, ρ are the kinematic viscosity and density of the fluid, respectively.

Find: Wall pressure at $X = 1$m and 4m.