20% 1) With the polynomial equation $(s+b)^3 + A(Ts+1) = 0$,

a) set up the characteristic equation in the form suited to the Evans root-locus method versus A and that versus T respectively.

b) is it possible to set it up versus b?, why or why not?

13% 2) For a feedback system with loop transfer function

$$G(s) = \frac{k}{(0.01s+1)(0.015s+1)\left(\frac{s^2}{10^2} + \frac{2(0.6)s}{10} + 1\right)},$$

use root-locus method to set gain k so that the dominant closed-loop poles have damping ratio of 0.4.

25% 3) For the system diagram shown below,

a) write down it's state space realization

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \quad ; \quad \mathbf{y} = C\mathbf{x}$$

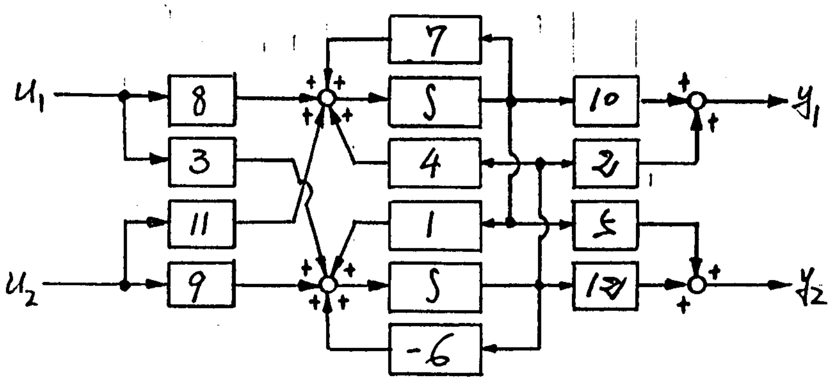
with $\mathbf{x} \in R^2$, $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ being the state, the input and the output, respectively.

b) is the system open-loop stable?

c) for the given output feedback control law

$$\mathbf{u} = G\mathbf{y} \quad ; \quad G = \begin{bmatrix} 1 & k \\ 0 & 2 \end{bmatrix},$$

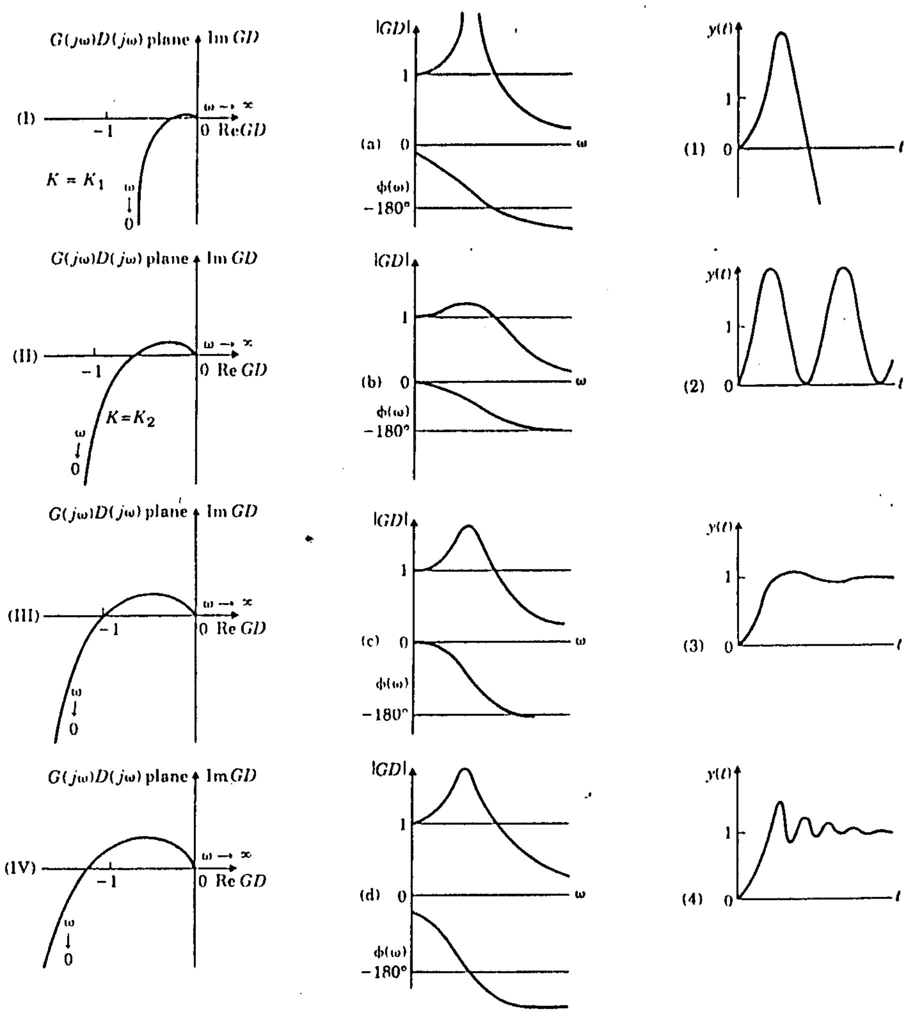
determine the range of k that leads to a stable closed-loop system.



12% 4) For the diagrams shown,

a) duplicate all the Nyquist plots and the Bode plots on your answer sheet with clearly specified gain margin and phase margin on each plot

b) correlate the Nyquist plots, the Bode plots and the step response charts, e.q. (I)=(a)=(1).



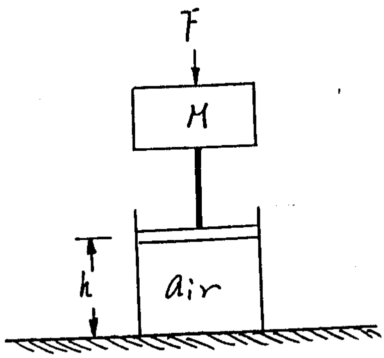
15% 5) The sketch below shows a mass supported by an air spring. Assume for the questions below that all compression and expansions are fast enough to be considered isentropic (i.e. PV^k = constant, $k = 1.4$).

a) Derive the differential equations (as a set of first-order equations) for the velocity and displacement of the mass.

b) Linearize the system equations for operation around $F = F_0$. Introduce incremental variable for the linearized equations.

c) Find the transfer function for velocity as a function of incremental force for the linearized system.

— All sub-questions are equally weighted —



15% 6) Consider a liquid tank sketched below. The tank has a straight wall with the cross sectional area (=A), and Q_{in} is constant ($Q_{in} = Q_0$). The outflow, Q_{out}, is proportional to the liquid level (h), and fluctuates around the steady state value (steady state outflow under $Q_{in} = Q_0$ and $V_0 = 0$) due to a sinusoidal motion of the piston. The piston is forced to change its velocity according to the relation $V(t) = V_0 \sin(\omega t)$, and the piston stroke is limited by x_s. The area of the piston is a. (We may neglect the piston mass and inertance effects of the liquid in the piston.)

a) Find the minimum value for ω which enables us to move the piston without saturating its position. Call that value ω_c.

b) Call the fluctuation in Q_{out} from the steady state value $\triangle Q_{out}$. Obtain $\triangle Q_{out}(t)$ as a function of V_0 and ω. Assume $\omega \geq \omega_c$.

c) For a fixed V_0, find the value for ω which causes the maximum amplitude of the outflow fluctuation.