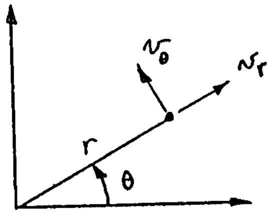
1. (20%) Consider a two-dimensional incompressible potential flow described by the velocity potential

$$\phi = (r^2 + \frac{1}{r^2}\cos 2\theta)$$



where r and θ are the polar coordinates illustrated.

(a) (10%) Determine an expression in polar coordinates for the corresponding stream function ψ, with the streamline through (r=1, θ=0) labeled ψ = 10.

(b) (10%) Determine expressions for the coordinates of ψ = 10 streamline, indicate the direction of the flow, the stagnation point, and sketch a few streamlines to illustrate the flow.

2. (20%) Two very long parallel plates of length $2L$ are separated by a distance b, as shown in Fig. 1. The upper plate moves downward at a steady rate V. An inviscid and incompressible fluid of density ρ fills the gap between the plates. Fluid is squeezed out between the plates, and since the flow is symmetrical, the velocity parallel to the plate at the center is zero. Assume that $b << L$ and that the velocity u parallel to the plate is essentially constant across the gap. Treat the flow as being one-dimensional and parallel to the x-axis.

(a) Show that the velocity at any point x from the center is approximately $u = Vx/b$. (5%)

(b) Noting that b changes with time and assuming that the pressure outside the plates is zero and the gravity effect on the fluid is negligible, first, obtain an unsteady Bernoulli equation. (5%) Secondly, find $\partial u/\partial t$. (2%) Thirdly, integrate the Bernoulli equation. (3%) Finally, obtain an expression for the pressure at any point x along the plate. (5%)

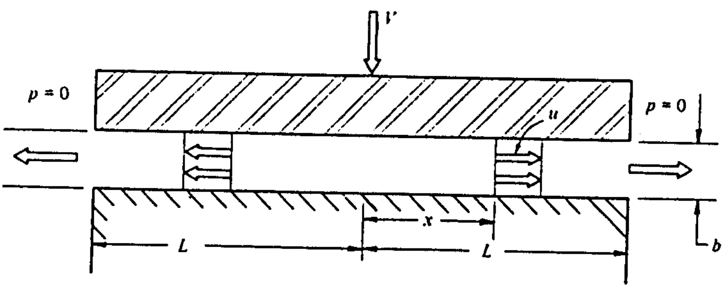
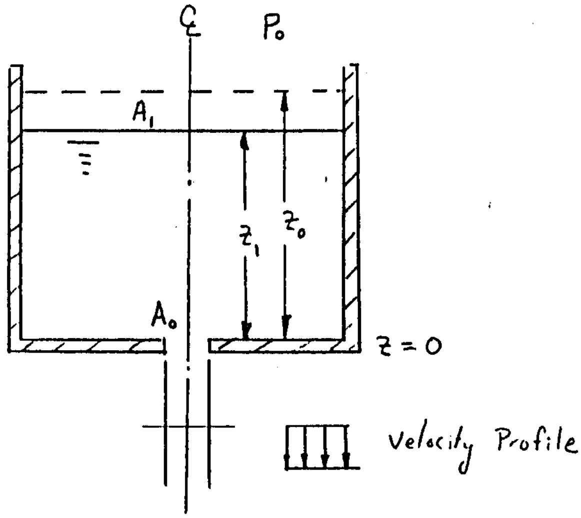


Fig. 1

3. (20%) Consider a cylindrical container as shown with a round orifice of area A_0 on the buttom and has a cross-sectional area of A_1 . The orifice was closed as $z = z_0$ with atmospheric pressure of P_0 . Withorifice being opened and the liquid level reaches z , neglecting the viscous effects and assuming the velocity profile in the jet is flat everywhere, answer the following questions:



(a) (10%) Treat the flow as quasi-steady, show that the jet velocity is

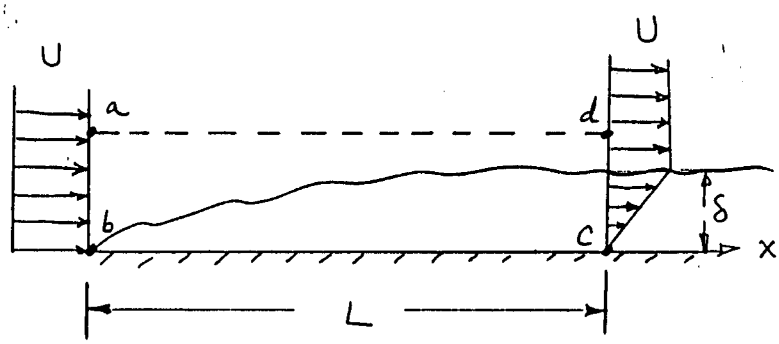
$$V = \sqrt{\frac{2 g z_1}{1 - \alpha^2}} \quad \text{where} \quad \alpha^2 = \frac{A_0}{A_1}$$

(b) (10%) Treat the flow as unsteady, assume the jet from orifice A is of uniform velocity V , show that

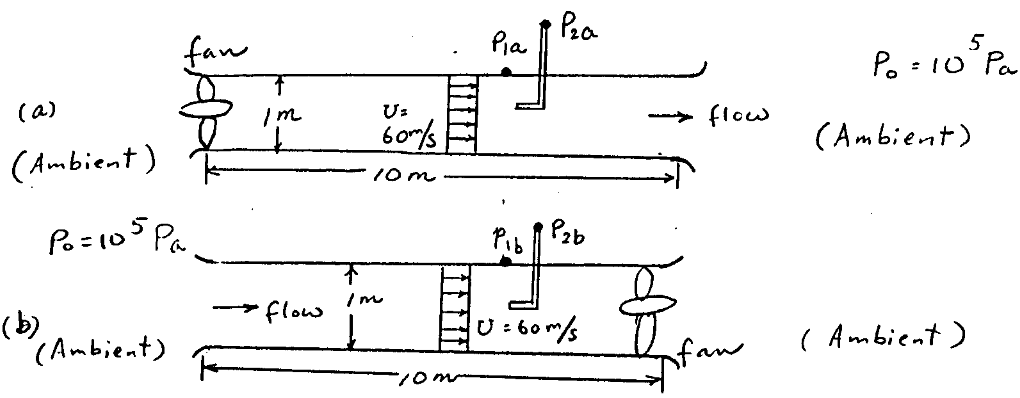
$$V = \sqrt{\frac{2 g z_1}{1 - 2\alpha^2}\left[1 - \left(\frac{z_1}{z_0}\right)^{\frac{1 - 2\alpha^2}{\alpha^2}}\right]}$$

$$\text{where} \quad \alpha = \frac{A_0}{A_1}$$

4. (20%) There is a steady viscous flow passing over the top of a flat plate as shown in the Figure. A boundary layer forms along the plate exerting a frictional drag force on the plate. Considering the velocity distributions on both sides of the control colume a b c d , and assuming that the pressure is constant through the boundary layer, determine : (1) the mass flux through the control surface a d , and (2) the drag force of the fluid on the plate.



5. (20%) Consider a uniform flow in a circular pipe driven by a fan which (a) is situated upstream, (b) is situated downstream. This circular pipe is placed in the ambient air which is at 1 atmospheric pressure.



P_{1a} , P_{2a} : Static and total pressures in the pipe "a", respectively, measured at 6m downstream of the fan.

P_{1b} , P_{2b} : Static and total pressures in the pipe "b", respectively, measured at 4m upstream of the fan.

P_o : Ambient pressure, 10^5 Pa

U : flow velocity, 60m/s

ρ : density of air, 1 kg/m^3

ν : kinematic viscosity fair, $16\text{x}10^{-5}$ m^2/s

The flow velocities in the two circular pipes are equal, U = 60m/s.

Assume that the boundary layer thickness developed on the wall is thin and is negligible in comparison with the diameter of the pipe. Find P_{1a} , P_{2a} , P_{1b} , P_{2b} .