1. Consider the following state space model

$$\dot{X} = AX + Bu \tag{1a}$$

$$Y = CX \tag{1b}$$

(a) 5% Explain the physical meanings of controllability and observability.

(b) 8% Design an example like Eq.(1) (i.e., choose some appropriate A, B, C matrices) such that following two conditions are satisfied simultaneously : (i) (A,B) is not controllable but is stabilizable. (ii) (A,C) is neither observable nor detectable. Check that your example truely satisfies above two conditions.

2. Consider following state-space model :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{2a}$$

$$y = [1 \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{2b}$$

(a) 3% Is this system controllable ?

(b) 3% Find the poles of this system.

(c) 5% If we use control law

$$u = -[k_1 \ k_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

, choose k_1 and k_2 such that the two new poles locate at $-2\omega_0$.

(d) 5% Find a similarity transformation

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

such that the new states

$$\begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} = T^{-1} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

has a state model

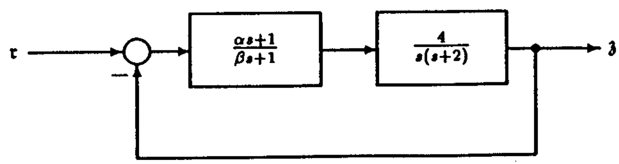
$$\begin{bmatrix} \dot{\zeta}_1 \\ \dot{\zeta}_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u \tag{3a}$$

$$y = [c_1 \ c_2] \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} \tag{3b}$$

Find the values of λ_1, λ_2, b_1, b_2, c_1, c_2.

(e) 4% According to the results of part (d), is this system observable, stabilizable, detectable ?

3.(33%) For the feedback control system shown, α and β are the design parameters. Using the root locus method to determine the effect of varying parameters. Select a suitable set of α and β so that the closed-loop settling time is less than 4 sec and the damping ratio of the dominante roots is greater than 0.6.



4. Carefully draw the Bode and Nyquist plot for the following systems, and conclude whether or not the loop is stable.

a). $G(s)=\frac{(s-10)}{(s^2+4s+8)}$ (8%)

b). $G(s)=\frac{10(s+1)}{s(s-10)}$ (7%)

5. Explain the following terminologies.

a). Frequency response (3%)

b). Non-minimum phase zeros and their effects on the closed loop performance (5%)

c). Resonant frequency (2%)

e). Nyquist stability criterion (5%)

f). Phase margin and gain margin (4%)