(14%) 1. If $F(x)=\int_0^x t(t-1)e^{-t^2}dt$,

(a) find the points of relative maxima and minima of $F(x)$.

(b) For what values of x is $F'(x)>0$, and for what values of x is $F'(x)<0$?

(14%) 2. The Fourier transform of $f(x)$ is defined as

$$F(\omega)=\int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx.$$

If $$f(x)=\begin{cases} e^{-x}, & x\geq 0 \\ 0, & x<0 \end{cases},$$

(a) find $F(\omega)$, and

(b) show that $|F(\omega)|$ decays as $1/\omega$ as $\omega\to\infty$.

(14%) 3. Solve the following differential equation:

$$(xe^y-1)dy+e^y dx=0$$

(14%) 4. Use the method of complex variables to evaluate the integral:

$$I=\int_0^{2\pi}\frac{d\theta}{\cos\theta+2}$$

(14%) 5. (a) Find a unit vector perpendicular to the plane $4x+2y+4z=-7$.

(b) A skew-Hermitian form of linear algebra operation is defined as $s=\bar{x}^T Ax$, where x is a vector, A is a square skew-Hermitian matrix, and $\bar{x}$ denotes the complex conjugate of x. For all possible choice of x, what will be value of s?

(15%) 6. Using the separation of variables method solve the following equation:

$$\begin{cases} u_t - u_{xx} = 0, & 0<x<1, \quad t>0 \\ u(x,0) = \sin 2\pi x, & 0<x<1 \\ u(0,t) = u(1,t) = 0, & t>0 \end{cases}$$

(15%) 7. Consider two coordinate frames S_A and S_B. Let $\bar{e}_i^A$, $\bar{e}_i^B$, i=1, 2, 3 be the orthogonal unit bases fixed to frames A and B with origin at O_A and O_B, respectively. In addition to translation, S_B is rotating in S_A with angular velocity $\overset{A-B}{\omega}(t)$. $\bar{R}(t)$ is the position vector from O_A to O_B. P is a point with position vector relative to O_B being $\bar{r}(t)$, and $\bar{r}(t)$ is expressed in terms of $\bar{e}_i^B$, i.e., $\bar{r}(t)=\sum_{i=1}^{3} r_i(t)\bar{e}_i^B$. Prove that the acceleration of point P relative to S_A is

$$\bar{a}_P = \ddot{\bar{R}}(t) + \overset{A-B}{\dot{\omega}} \times \bar{r}(t) + \overset{A-B}{\omega} \times (\overset{A-B}{\omega} \times \bar{r}) + 2\overset{A-B}{\omega} \times \left(\sum_{i=1}^{3} \dot{r}_i \bar{e}_i^B\right) + \sum_{i=1}^{3} \ddot{r}_i \bar{e}_i^B$$

(Hint: $\frac{d}{dt}\bar{e}_i^B$ relative to frame S_A is $\overset{A-B}{\omega}(t) \times \bar{e}_i^B$.)