1) In the two-dimensional incompressible case, let $\rho$, $p$, and $\bar{V}$ are density, pressure, and velocity. The Novier-Stokes equation is given as

$$(1)\quad \begin{cases} \rho \dfrac{D\vec{V}}{Dt} = \rho f - \ grad\, p + \mu \nabla^2 \vec{V} \\ div\, \vec{V} = 0 \end{cases}$$

where $\dfrac{D}{Dt}$ is material derivative and $\nabla^2$ is the Laplace operator.

(a)(6%) If $\rho =$ constant and $f =$ grad $\varphi$, prove that (1) implies the vorticity equation

$$\frac{Dw}{Dt} = (w \bullet grad)\vec{V} + \frac{\mu}{\rho}\nabla^2 w$$

where $w = \text{curl } \vec{V}$

(b) (7%)If $\ell$, $\nu / \ell^2, U$, and $\rho_0\, \nu U / \ell$ where $\nu \equiv \mu / \rho_0$ are length, time, volocity, and pressure scales, show that in dimensionless form the govering equation (1) Can be written

$$div\, \vec{V} = 0, \quad \vec{V}_t + \text{Re}(\vec{V} - \text{grad})\vec{V} = -\text{grad}\, p + \nabla^2 V$$

where $\text{Re} \equiv U\ell / \nu$.

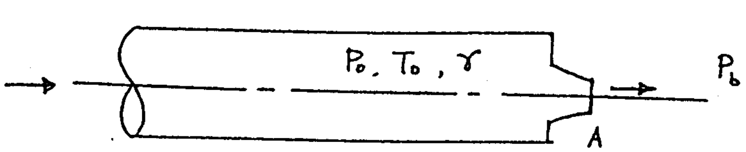
(c) (7%)For flows with small Reynolds number, that Re<<1, show that

$$\nabla^2 p = 0, \quad \frac{\partial}{\partial t}(\nabla^2 \vec{V}) = \nabla^2\left(\nabla^2 \vec{V}\right).$$

2.) A flow of stagnation pressure $P_0$ and stagnation temperature $T_0$ is discharging through a converging nozzle of exit area A to a back pressure of $P_b$. Assume steady flow of perfect gas and specific heat ratio as $\gamma$.

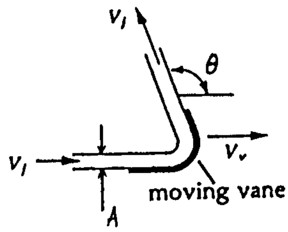
(a) Derive the mass flow rate $\dot{m}$ at nozzle exit as function of $P_0$, $T_0$, $P_b$, A and $\gamma$. (10%)

(b) Prove that the mass flow rate is maximum when the nozzle is choking. (10%)



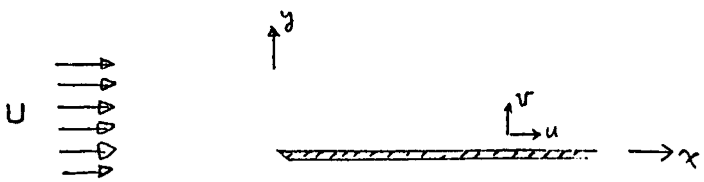
3). (a) A jet of water with density $\rho$ striking a curved vane, which turns the jet through an angle of $\theta$, as shown in Fig. a. The vane is moving to the right at a velocity of $V_v$, and the jet has a cross-sectional area $A$. Develop an expression for forces, $F_x$ and $F_y$, exerted on the vane, assuming no frictional loss. (15%)

(b) When the vane is stationary, determine $F_x$ and $F_y$. (5%)




(a)

4). a. 下圖爲在無限大空間之均勻流場中的一塊平板。試繪圖說明在平板上方的流場內，那一些區域是流體黏性會發生作用的區域，爲什麼? (3%)




b. 說明什麼情形下，流體黏性發生作用區域會變成邊界層。(1%)

c. 什麼情形下，邊界層內的流體會由層流變成紊流。並繪圖說明邊界層內，層流區及紊流區的速度分佈示意圖，並說明層流與紊流層之速度有何不同，爲什麼不同。(8%)

d. 設 $\frac{\partial p}{\partial x}=0$，試由下面的Navier Stokes方程式，使用數量級大小比較法，証明邊界層中，$\frac{\partial p}{\partial y}\sim 0$。(8%)

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]$$

5) The application of the hydrodynamic theory of Couette flow in the study of lubrication can be illustrated best by an example of the slipper bearing (Fig. a). The gap between the slipper block and the bearing guide is always much smaller than the length of the block and is filled with a lubricant, usually oil. In order to make the motion steady, a system of coordinates is chosen in which the slipper block is stationary and the bearing guide moves with a velocity $-U$ (Fig. b). For a steady, two-dimensional flow and a small inclination angle $\alpha$, we have $v << u$, $\frac{\partial}{\partial t} = 0$. $u$ is the velocity component in the x direction and $v$ is the velocity component in the y direction. Please answer the following porblems: 20%

a) Find out the governing equations and boundary conditions. (10%)

b) Find out the mass flow rate. (10%)

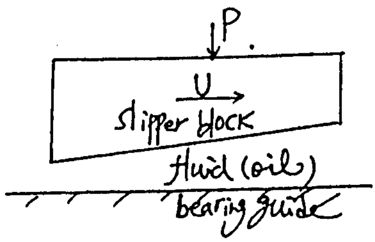



Fig. a

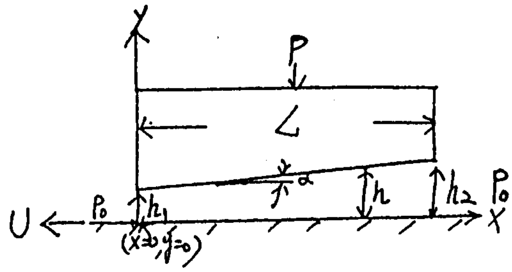



Fig. b

$\left( \frac{h}{L} \approx 10^{-3},\ Re = \frac{\rho U L}{\mu} < 2.5 \times 10^{4} \right)$