1) In the two-dimensional incompressible case, let ρ, p, and $\vec{V}$ are density, pressure, and velocity. The Novier-Stokes equation is given as

$$(1)\quad \begin{cases} \rho\dfrac{D\vec{V}}{Dt} = \rho f - \ grad\, p + \mu\nabla^2\vec{V} \\ div\,\vec{V} = 0 \end{cases}$$

where $\dfrac{D}{Dt}$ is material derivative and ∇^2 is the Laplace operator.

(a)(6%) If $\rho =$ constant and $f =$ grad φ, prove that (1) implies the vorticity equation

$$\frac{Dw}{Dt} = (w \bullet grad)\vec{V} + \frac{\mu}{\rho}\nabla^2 w$$

where $w = \mathrm{curl}\ \vec{V}$

(b) (7%)If ℓ, ν/ℓ^2, U, and $\rho_0\,\nu U/\ell$ where $\nu \equiv \mu/\rho_0$ are length, time, volocity, and pressure scales, show that in dimensionless form the govering equation (1) Can be written

$$div\ \vec{V} = 0, \quad \vec{V}_t + \mathrm{Re}(\vec{V} - \mathrm{grad})\vec{V} = -\mathrm{grad}\,p + \nabla^2 V$$

where $\mathrm{Re} \equiv U\ell/\nu$.

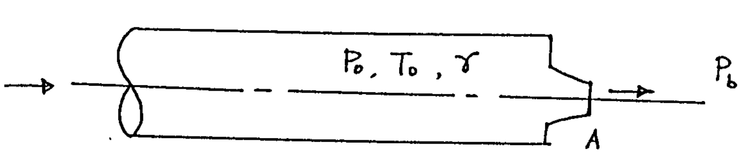
(c) (7%)For flows with small Reynolds number, that Re<<1, show that

$$\nabla^2 p = 0, \quad \frac{\partial}{\partial t}(\nabla^2\vec{V}) = \nabla^2\left(\nabla^2\vec{V}\right).$$

2.) A flow of stagnation pressure P_0 and stagnation temperature T_0 is discharging through a converging nozzle of exit area A to a back pressure of P_b. Assume steady flow of perfect gas and specific heat ratio as γ.

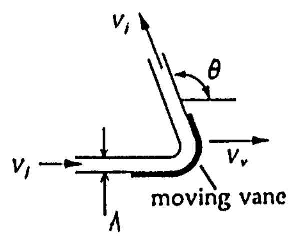
(a) Derive the mass flow rate $\dot{m}$ at nozzle exit as function of P_0, T_0, P_b, A and γ. (10%)

(b) Prove that the mass flow rate is maximum when the nozzle is choking. (10%)



3). (a) A jet of water with density ρ striking a curved vane, which turns the jet through an angle of θ, as shown in Fig. a. The vane is moving to the right at a velocity of V_v, and the jet has a cross-sectional area A. Develop an expression for forces, F_x and F_y, exerted on the vane, assuming no frictional loss. (15%)

(b) When the vane is stationary, determine F_x and F_y. (5%)



(a)

4) Consider a thin airfoil of infinite span in a 2 D steady incompressible inviscid flow. The camber line is an almost flat parabolic arc:

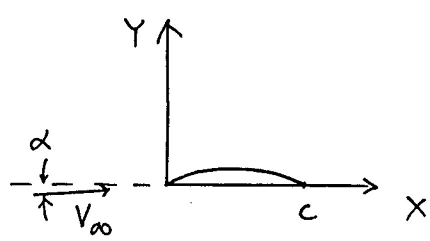
$$Y(x) = a\,x(c-x)$$

where c is the chord length and a is a small constant. The free stream speed is V_∞ and the angle of attack is α.

(a) Find the lift distribution $\ell(x)$ on the airfoil as a function of x. (10%)

(b) Prove that your solution satisfies the Kutta condition. (5%)

(c) Find the center of pressure Xcp. (5%)



5) A wedge-shaped wing section is flying supersonically as shown in the sketch.

(a) Derive the wave drag formula using linearized potential flow model. (10%)

(b) Is Kutta condition satisfied? State your reason why Kutta condition should be, or should not be satisfied? (5%)

(c) Suggest your way of reducing the wave drag. (5%)