Engineering Mathmatics

Entrance Exam

(10%) 1. Solve the initial value problem:

$$y'' - 2y' + y = xe^x + \sin x,$$

$$y(0) = 0, \quad y'(0) = 1.$$

(12%) 2. Let $C_i(s)$ be the Laplace transform of $c_i(t)$. Find $\lim_{t\to\infty} c_i(t)$, and $\frac{d}{dt}c_i(t)|_{t=0+}$ for the following $C_i(s)$

(a) $C_1(s) = \frac{s}{s+2}$,

(b) $C_2(s) = \frac{2s+1}{s^2+2s+3}$,

(c) $C_3(s) = \frac{3s+2}{s^2(s^2+2s+3)}$,

(d) $C_4(s) = \frac{3s^2+2s+1}{s(s^2+2s+3)(s+4)}$,

(e) $C_5(s) = \frac{2s+5}{s^3+2s^2-5s+5}$,

(f) $C_6(s) = \frac{3s^2+2s+1}{(s^2+3)(s+1)}$.

(14%) 3. Consider the complex-valued function:

$$f(z) = \frac{\cos(1/z)}{z^3(z^2+4)}.$$

(a) What are the singular points of $f(z)$? Also *classify* the singularities.

(b) Evaluate $\oint_C f(z)dz$ where $C : |z-2i| = 1$ oriented counterclockwise.

(15%) 4.

(a) Find the derivative of the function $x = g(y)$ inverse to the function $y = f(x) = x + x^5$ at $y = 2, x = 1$.

(b) Find the slope of the tangent to the circle $(x-3)^2+(y+1)^2 = 37$ at the point $x = 2, y = 5$.

(c) Find the derivative, at $x = 0, y = 0$, for the function defined near this point by the equation $x^2 - y^2 = 0$.

(d) Let C be a curve leading from $(1,1,1)$ to $(2,4,6)$. Compute $\int_C y\,dx + (x+z^2)dy + 2yz\,dz$.

(15%) 5.

(a) Consider the linear space which is spanned by the following vectors

$$\underset{\sim}{a}_1 = (2,3,1,2), \quad \underset{\sim}{a}_2 = (3,4,5,7), \quad \underset{\sim}{a}_3 = (1,1,2,1).$$

Construct a set of orthogonal unit basis for this linear space.

(b) Consider the following linear algebra equation

$$\underset{\sim}{A}\underset{\sim}{x} = \underset{\sim}{y}$$

where $\underset{\sim}{A} \in R^{n\times m}$, $\underset{\sim}{y} \in R^{n\times 1}$ are given. Give the conditions under which $\underset{\sim}{x}$ exists. (Note that $\underset{\sim}{A}$ may not be a square matrix.)

(15%) 6.

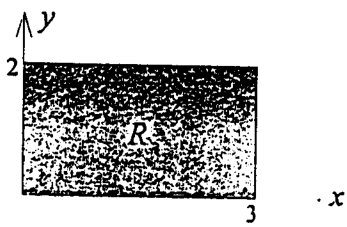
(a) If $\underset{\sim}{u}$, $\underset{\sim}{v}$ are two vectors, derive the *triangle inequality*:

$$\|\underset{\sim}{u} + \underset{\sim}{v}\| \le \|\underset{\sim}{u}\| + \|\underset{\sim}{v}\|,$$

where $\|\ \|$ denotes the Euclidean norm of a vector. (Hint: Schwarz inequality: $|\underset{\sim}{u}\cdot\underset{\sim}{v}| \le \|\underset{\sim}{u}\|\|\underset{\sim}{v}\|$)

(b) Given a scalar field $u(x,y,z,t) = x^3y^2zt$ and a vector field $\underset{\sim}{v}(x,y,z) = x^2y\underset{\sim}{i} - 3\underset{\sim}{j}$ in Cartesian coordinate system, find grad$u$, div$\underset{\sim}{v}$, and curl$\underset{\sim}{v}$.

(c) Verify the *Divergence Theorem* for the vector field $\underset{\sim}{v}$ in (b) over the two-dimensional region R, which is a rectangle shown below.



(19%) 7. Solve the following partial differential equation

$$\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$$

with the initial conditions $u(x,0) = x^3$ over $0 < x < \ell$ and the end conditions

$$u(0,t) = 0, \quad u(\ell,t) = 0, \quad \text{for} \quad 0 < t < \infty.$$

Can you find the solution if you change the initial conditions to $u(x,0) = x^4$ over $0 < x < \ell$? Why?