1. A fluid with density ρ, viscosity μ, and velocity V flows through a pipe with diameter D and length L. The frictional pressure loss Δp in the pipe flow can be expressed by other variables:

$$\Delta p = \Delta p(\rho, V, \mu, D, L, a) \quad (1)$$

where a is the speed of sound. You are asked to perform a dimensional analysis for Eq. (1) using the dimensional system, mass-length-time, and assuming

$$\Delta p = C_1 \rho^{b_1} V^{b_2} \mu^{b_3} D^{b_4} L^{b_5}, \quad C_1 = \text{const.}$$

in which $b_i, i = 1, 2, \ldots, 5$, are to be determined. Note that the equation you obtained should be expressed in a dimensionless form. (15%) Moreover, identify some common parameters (their common names) (5%).

2. Consider an air flow in a circular pipe, see the figure. Flow passes the screen at $x = 0$ which results into momentum loss.

Assume: (i) At x = -1d, flow is uniform in velocity distribution with the velocity U_1 and the pressure P_1 measured.

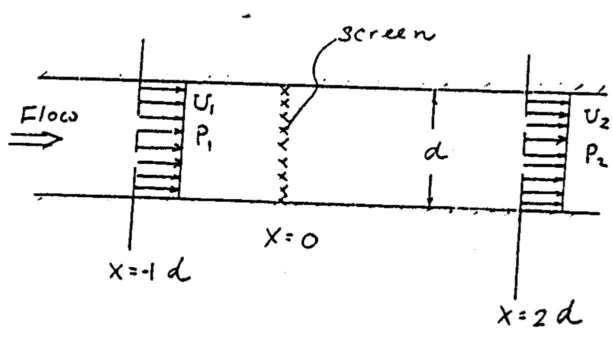
(ii) At x = 2d, flow is uniform in velocity distribution with the velocity U_2 and the pressure P_2 measured.

Questions: (20%)

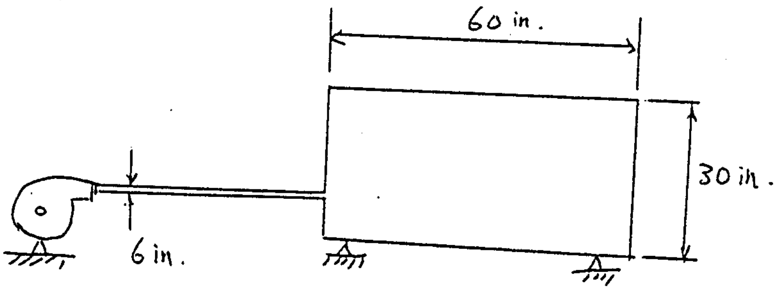
(i) Is $U_1 = U_2$, $U_1 > U_2$, or $U_1 < U_2$? why?

(ii) Is $P_1 = P_2$, $P_1 > P_2$, or $P_1 < P_2$? why?

(iii) Can you apply the Bernoulli's equation for flow in the entire region -1d < x < 2d?



3. An air compressor is used to pressurize an initially evacuated tank. The tank is 30 inches in diameter and 60 inches long. The supply line is 6 inches in diameter and conveys a flow of 5 ft/s. The air compressor's output pressure and temperature are constant at 50 psia and 90° F. The tank temperature of 70° F is also constant. Calculate the time required for the tank pressure to reach 15 psia. (20%)



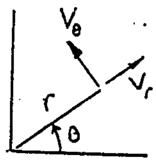
4. For the steady incompressible flat-plate laminar boundary-layer flow, the velocity profile is assumed as follows:

$$\frac{u}{U}=\frac{3}{2}\left(\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

By using the integral momentum analysis, compute (a) $(\frac{\theta}{x})\sqrt{Re_x}$, (b) $(\frac{\delta^*}{x})\sqrt{Re_x}$, (c) $(\frac{\delta}{x})\sqrt{Re_x}$, (d) $C_f\sqrt{Re_x}$ and (e) $C_D\sqrt{Re_x}$ (20%)

θ: momentum thickness, δ^*: displacement thickness, δ: boundary-layer thickness
C_f: friction coefficient, C_D: drag coefficient, Re_x: Reynolds number
U: freestream velocity

5. Consider a steady and inviscid flow with negligible gravity. Answer the following questions: (20%)



(polar coordinates)

a) The stream function of a uniform stream (freestream velocity = U) passing a solid body is expressed as

$$\Psi = Ur(1-\frac{a^2}{r^2})\sin\theta, \qquad a = \text{constant}$$

What would be the shape of this solid body if $\Psi = 0$ is chosen to be the surface stream function?

b) What would the flowfield be modified if an additional term is added to the previous stream function to result in a new form

$$\Psi = Ur(1-\frac{a^2}{r^2})\sin\theta + \frac{G}{2\pi}\ell n\, r, \qquad G < 4\pi aU$$

Does the region of the flowfield $r > a$ become rotational after this modification? State your reasons about why you made such an assertion.

c) The radial and circumferential velocity components can be expressed as

$$V_r = \frac{1}{r}\frac{\partial \Psi}{\partial \theta}$$
$$V_\theta = -\frac{\partial \Psi}{\partial r}$$

Determine V_r and V_θ at $r = 2a$ and find the line integral around a circle of $r = 2a$

$$\oint_{r=2a} rV_\theta d\theta = ?$$

Is there any physical meaning associated with this line integral?

d) Determine the lift force exerting on this body as the stream function in question b) is taken to represent the flowfield.