1. As shown in Fig. 1, the motion of a particle P can be represented by

$$r(\theta) = \frac{p}{1 + e\cos\theta}$$

where p and e are constant.

(a) Determine the velocity and acceleration of the particle at the position (r, θ) using $\mathbf{e}_r$ and $\mathbf{e}_\theta$ as unit vectors. (10%)

(b) Determine the velocity and acceleration of the particle at the position (r, θ) using unit tangent vector $\mathbf{e}_t$ and unit normal vector $\mathbf{e}_n$. (10%)

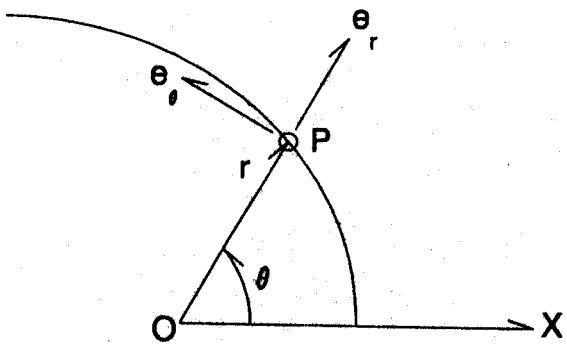

Figure 1

2. As shown in Fig. 2, IJK are unit vectors of a fixed frame and $\mathbf{e}_r\mathbf{e}_\theta\mathbf{e}_\phi$ are unit vectors of spherical coordinates.

(a) Determine $\mathbf{e}_r\mathbf{e}_\theta\mathbf{e}_\phi$ in terms of θ, ϕ and I, J, K. (5%)

(b) Determine I, J, K in terms of θ, ϕ and $\mathbf{e}_r\mathbf{e}_\theta\mathbf{e}_\phi$. (5%)

(c) If $\theta = \theta(t)$ and $\phi = \phi(t)$. Determine the angular velocity of $\mathbf{e}_r\mathbf{e}_\theta\mathbf{e}_\phi$. Express it both in $\mathbf{e}_r\mathbf{e}_\theta\mathbf{e}_\phi$ frame and IJK frame. (10%)

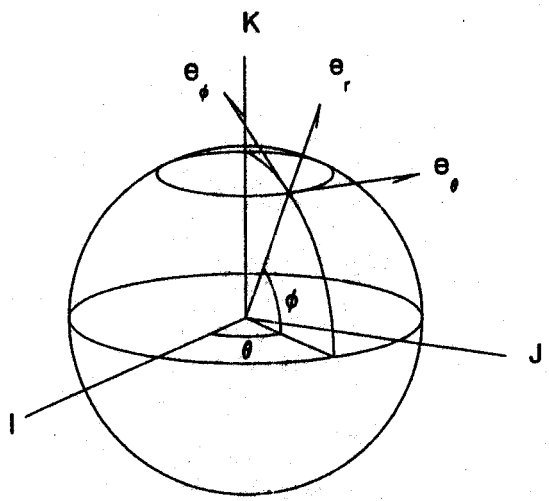

Figure 2

3. Consider Fig. 3. A square block of mass m and length h is pushed off the top of a hemisphere of radius R by an impulse force F. Assume that there is no friction between the block and the hemisphere, find the angle θ where the block leaves the surface of the hemisphere. (20%)

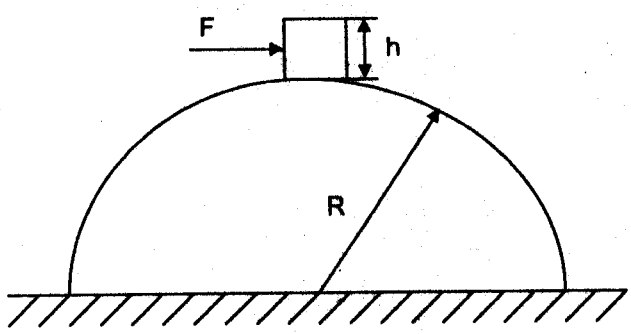

Figure 3

4. A boy, holding horizontally a flywheel of with constant angular velocity pointing to his left, is sitting on a chair, which can rotate freely about its vertical axis as shown in Fig. 4. Discuss what would happen if the boy rotates the flywheel clockwise as viewed from the boy's side. (20%)

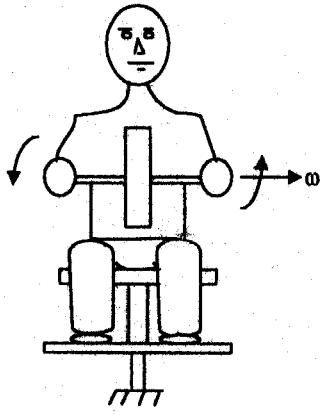

Figure 4

5. A homogeneous beam of length L and mass m_2 is pinned to a colar of mass m_1, which slides frictionlessly on a horizontal pipe as shown in Fig. 5. Find the accelaration of the colar at the moment when the beam is released from an angle θ relative to the vertical position. (20%)

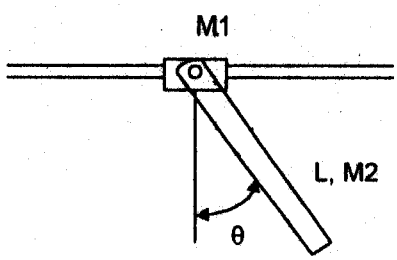

Figure 5