(20%) 1. Find the general solution of

$$\frac{d^2 y}{d x^2}+3\frac{d\,y}{d\,x}+2y=6e^x,$$

(15%) 2. Given a real valued function , ϕ, that ϕ : $R^3 \rightarrow R$ and

$$\phi(x,y,z)=x^2+3y^2-z,$$

determine the equation of a plane tangent to the surface of $\phi(x,y,z)=3$ at the point (1 ,1 ,1).

(15%) 3. let $\tilde{e}_1=\begin{bmatrix}0\\1\\1\end{bmatrix},\tilde{e}_2=\begin{bmatrix}1\\-1\\2\end{bmatrix},\tilde{e}_3=\begin{bmatrix}2\\1\\-1\end{bmatrix}$

(a)show that B=$\{\tilde{e}_1,\tilde{e}_2,\tilde{e}_3\}$ is a basis for the space

$$C^3=\left\{v:\ v=\begin{bmatrix}a\\b\\c\end{bmatrix},a,b,c\in R\right\}$$

(b) Let $\left\{e_1=\begin{bmatrix}1\\0\\0\end{bmatrix},e_2=\begin{bmatrix}0\\1\\0\end{bmatrix},e_3=\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}=D$ be the standard basis for C^3,

and

$x=x^i e_i=\tilde{x}^i\tilde{e}_i$ so that $[x]_D=p[x]_B$, $[x]_D$ and $[x]_B$ are the coordinates of x represented by the basis D or B. Find P.

(15%) 4. Evaluate $I=\int_{-\infty}^{\infty}\frac{\cos(mx)}{a^2+x^2}\,dx\ \ (m\geq 0,a\rangle 0)$

(15%)5. At a neighborhood ,N, of $(x_0, y_0) = (0,0)$, give the transformation,

$$\xi(x,y)\ ,\ \eta(x,y):$$

$$\xi(x,y) = x + y^4 + 2x^2 y$$

$$\eta(x,y) = y + x^3 + x y^2$$

(a) Please explain why there exists an inverse transformation around the point (0,0)

$$x = x(\xi, \eta)\ ,\ y = y(\xi, \eta)$$

(b)Please Compute ξ_x, ξ_y, η_x, η_y and x_ξ, x_η, y_ξ, y_η.

(20%) 6. Compute the solution of the boundory value problem

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}\ ,\ 0 < x < 1\ ,\ t > 0$$

$$u(0,\ t) = u(1,\ t) = 0,\ t > 0$$

$$u(x,0) = 1 - \cos(2\pi x)\ ,\ 0 < x < 1$$