1. A uniform heating wire is put inside a water tank horizontally as shown in Fig. 1. With the wire being heated, it is observed that the vapor bubble on the wire always moves along the wire for a distance on either direction before it moves away from the wire. Please explain this phenomenon physically and mathemmatically. (20%)

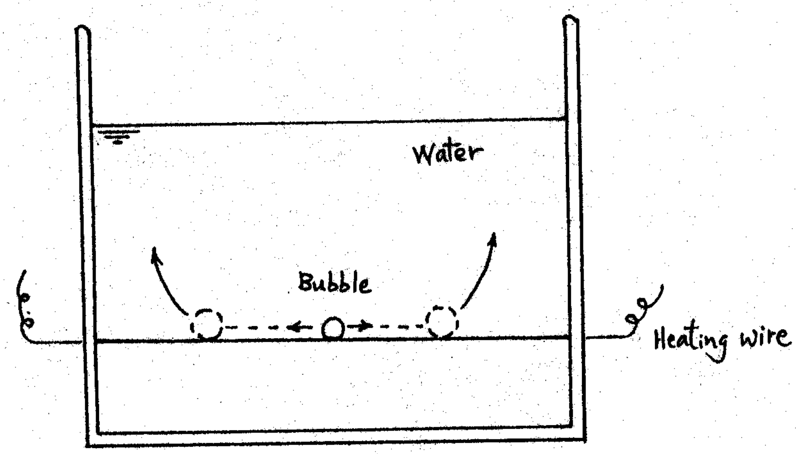

Fig. 1

2. A city has a fire truck whose pump and hose can deliver 1000 gal/min of water with a nozzle velocity of 120 ft/sec. The tallest building in the city is 100 ft high. The firefighters hold the nozzle at an angle of 75° from the ground. Find the minimum distance, d, the firefighters must stand from the building to put out a fire on the roof without the aid of a ladder (see Fig. 2). The firefighters hold the hose 5ft above the ground. Assume that the water velocity is not reduced by air resistance. (Note that you need to write an expression for d. 1 gal/min=1/7.481ft^3/min) (20%)

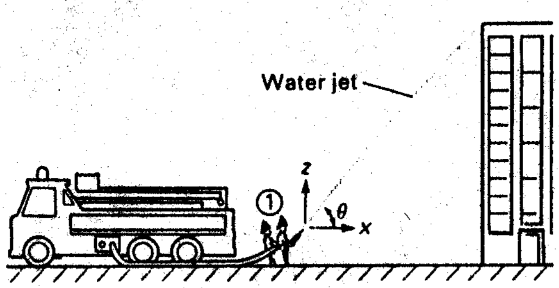

Fig. 2

3. (a) For a two-dimensional irrotational flow, let Φ be the velocity potential and Ψ be the stream function, and $F(z) = \Phi(x, y) + i\Psi(x, y)$ be the complex potential. The complex potential for a source of strength m located at $z = z_0$ will be

$$F(z) = \frac{m}{2\pi}\log(z - z_0) \text{。}$$

Please draw the graph of its streamlines and explain the physical meaning of m. (10%)

(b) Write down the complex potential for a source and a sink of equal strength m positioned at $x = \pm a$ along the x axis. Using the identity,

$$\tan^{-1} A - \tan^{-1} B = \tan^{-1}\left(\frac{A - B}{1 + AB}\right),$$

Find out the equation of streamlines. (10%)

4. Suppose a viscous flow with no body force has a velocity distribution

$$\vec{v}(x,y,z) = (3z + 4x)\vec{i} + (-5y)\vec{j} + (-2x + z)\vec{k}$$

where (x,y,z) is the Cartesian coordinates. The molecular viscosity is μ and is assumed constant. A fluid element of volume dV is located at (1,1,1).

(a) What is the angular velocity of this fluid element ? (5%)

(b) What is the rate of change of volume of this fluid element ? (5%)

(c) Suppose the fluid element has a surface described by

$$d\vec{s} = |ds|\vec{n} = |ds|(\vec{i} + \vec{j}) / \sqrt{2}$$

where $\vec{n}$ is the unit surface normal and $|ds|$ is the area. Find the viscous force per unit area acting on this surface by the fluid outside the element. (10%)

5. Consider a steady viscous flow over the top of a flat plate as shown in Fig. 3. Assume the velocity distribution is uniform and the boundary layer thickness is zero at X=0, while at X=L the velocity distribution varies linearly from 0 at Y=0 to U_∞ at Y= δ and the boundary layer thickness is δ. Suppose the pressure keeps constant throughout the flow field. Answer the following questions：
   (a) Is there any mass flux across the boundary layer ? If no, why ? If yes, determine this mass flux.(8%)
   (b) What force(s) causes the velocity distribution to change from uniform to linear ? Calculate this force if it exists.(12%)

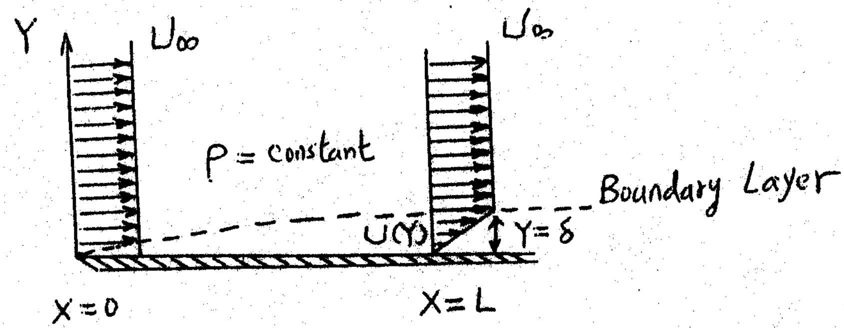

Fig. 3