1. A viscous flow of viscosity μ is given by the stream function
(20%)

$$\psi = -Axy, \qquad A > 0.$$

Assume that there is no body force. You are asked to obtain a relation between the pressure, density, A, x, and y.

2. Show that
(20%)

$$\phi = Ar^{\frac{\pi}{\alpha}} cos(\frac{\pi}{\alpha}\theta)$$

is a velocity potential of an inviscid flow, where A and α are constants, and r, θ are the polar coordinates.

3. The two-dimensional steady incompressible uniform flow over a flat plate will produce
(20%) a boundary layer flow. The boundary flow has the following governing equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial p}{\partial y} = 0$$

where u, v are the velocity components in x and y-directions, respectively, p is the pressure, and ρ is the density.

a. Derive the von Karman momentum integral equation for the boundary layer flow. (8%)

b. Assume linear variation of velocity within the boundary, say $\frac{u}{U} = \frac{y}{\delta}$, where U is the free stream velocity, $\delta = \delta(x)$ is the boundary layer thickness. Prove that the boundary thickness satisfy the following relation

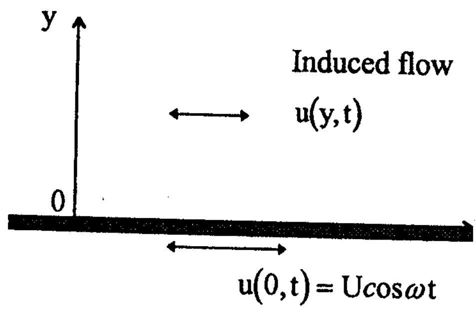
$$\delta = \delta(x) = \frac{3.46x}{\sqrt{Re_x}}$$

where $Re_x = \frac{\rho U x}{\mu}$ (8%)

c. Draw a schematic diagram to show a more correct velocity distribution within the boundary layer, and discuss why the above boundary layer thickness of item(2) is underestimated. (The exact solution is $\delta = \frac{5x}{\sqrt{Re_x}}$.) (4 %)

(背面仍有題目,請繼續作答)

4. Consider viscous flow over an oscillating wall of infinite length, as shown
(20%)



(a) Write down the governing equations and boundary conditions to describe the problem.

(b) Find the final solution of the velocity distribution $u(y,t)$ in the flow field.

(c) Show that for the height δ above which the induced flow velocity is less than 0.01U.

$$\delta = 6.4\left(\frac{\omega}{\nu}\right)^{\frac{1}{2}}$$

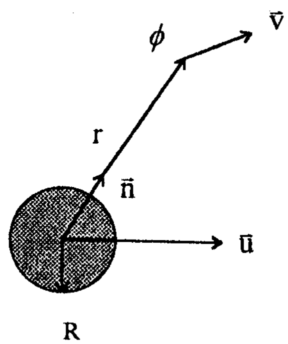
Note : The fluid kinematic viscosity is ν .

5. An infinite cylinder of radius R moving perpendicular to its axis with velocity
(20%) $\bar{u}$ in an incompressible ideal fluid.

(a) Determine the velocity potential ϕ of flow past the cylinder.

(b) Find the velocity distribution $\bar{v}$ in the flow field.

(c) Find the pressure distribution p on the cylinder surface.



r : distance from center of cylinder to a position in flow field.

$\bar{n}$: outward normal vector of cylinder surface along r.