1. The two-dimensional velocity potential of the inviscid flow field of an uniform flow over a rotating circular cylinder is

$$\phi = -V_\infty r \cos\theta - \frac{\chi \cos\theta}{r} + \frac{\Lambda}{2\pi}\theta$$

where r and θ are the two-dimensional polar coordinates with the origin locates at the cylinder center line as shown in the attached figure, V_∞ is the free stream velocity, χ is the doublet strength, and Λ is the vortex strength. The cylinder surface is at the radius of $r = \sqrt{\frac{\chi}{V_\infty}}$.

a. Prove that the flow field is irrotational. (8%)

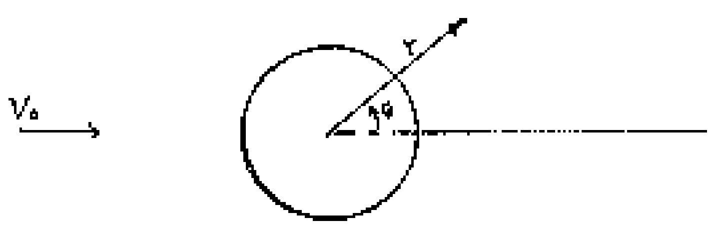
b. For a positive and negative Λ, draw a schematic diagram to indicate the corresponding effective force directions induced by the vortex and explain (12%).

Hint: For the cylindrical coordinate system the gradient and curl operator can use the following equations.

$$\nabla f = \frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{\partial f}{\partial z}\vec{e}_z$$

$$\nabla \times \vec{A} = \begin{bmatrix} \vec{e}_r & r\vec{e}_\theta & \vec{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & rA_\theta & A_z \end{bmatrix}$$

$$\vec{A} = A_r\vec{e}_r + A_\theta\vec{e}_\theta + A_z\vec{e}_z$$

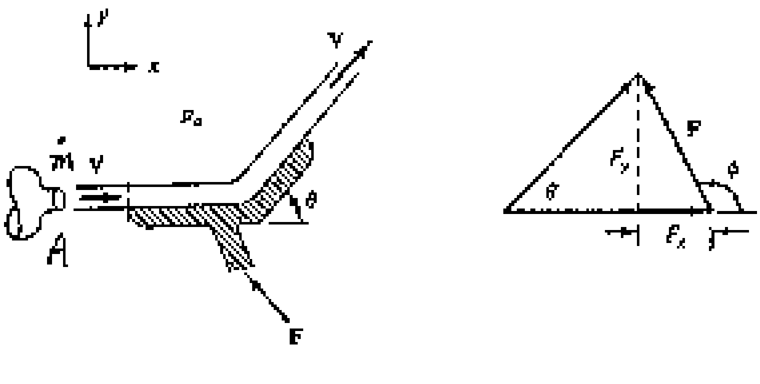


2. Consider a polar coordinate system, (r, θ). Given three stream functions

$$(i)\ \psi_1 = V_\infty r \sin\theta;\quad (ii)\ \psi_2 = \frac{\Lambda}{2\pi}\theta;\quad (iii)\ \psi = \psi_1 + \psi_2.$$

(a) please clearly explain the flows corresponding to these three stream functions. (6%)
(b) Compute the velocities corresponding to the stream functions ψ, ψ_1, ψ_2.(6%)
(c) Write down the streamline equation corresponding to the third kind flow (iii), and clearly sketch the streamlines. Note that you need to indicate where is the stagnation point.(8%)

3. (20%)
As shown in Fig. 1, a fixed vane turns a water jet of velocity V, mass flow rate $\dot{m}$, and area A, through an angle θ without changing its velocity magnitude and mass flow rate. Assume that the flow is steady, pressure is P_a everywhere, and friction on the vane is negligible. (a) Find the components F_x and F_y of the applied vane force. (b) Find expressions for the magnitide F and the angle ϕ in terms of given parameters. (c) Plot the curves for $\frac{F}{\dot{m}V}$ and ϕ versus θ in a figure.



(a) (b)

Fig. 1.

(背面仍有題目,請繼續作答)

4. (20%)

(a) Show that a lifting line of span b and of constant circulation Γ, the drag induced by the starting vortex filament alone is expressed by

$$D=\frac{\rho\Gamma^2}{2\pi}\left[\sqrt{1+\left(\frac{b}{\ell}\right)^2}-1\right]$$

where ℓ is the distance between starting vortex and the lifting line.(10%)

(b) In finite-wing circulation, this drag is neglected. Why?(10%)

5. (20%)

The induced drag of a finite wing can be found as

$$D_i=\frac{\rho}{4\pi}\int_{-b/2}^{b/2}\oint_{-b/2}^{b/2}\Gamma(y)\frac{d\Gamma}{dy_1}\frac{dy_1dy}{(y-y_1)}$$

where $\Gamma(y)$ is the circulation distribution along the span $(-b/2\le y\le b/2)$.

If the circulation is expressed as

$$\Gamma(y)=Ub\sum_n A_n\sin n\theta\ ,\quad y=\frac{b}{2}\cos\theta$$

show that

$$D_i=\pi\frac{\rho U^2b^2}{8}\sum_{n=1} nA_n^{\,2}$$

note that

$$\int_0^\pi \frac{\cos n\theta_1}{\cos\theta_1-\cos\theta}d\theta_1=\pi\frac{\sin n\theta}{\sin\theta}$$

ρ: air density

b: span

U: freestream velocity