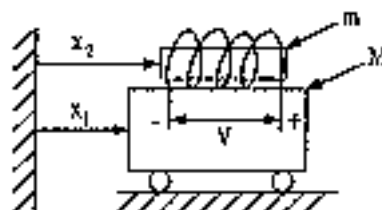


1. (10%) Consider a standard mass-spring-damper system,

$$m\ddot{x} + c\dot{x} + kx = u,$$

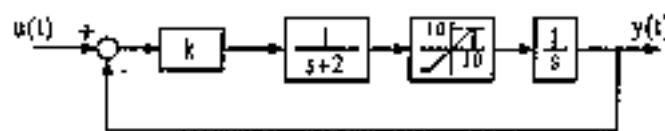
where  $m$ ,  $c$ ,  $k$  are mass, damping coefficient and spring constant, respectively, and  $x$  and  $u$  are mass position and input force. Is the condition,  $x = x_d > 0$ ,  $\dot{x} = 0$  and  $u = 0$ , stable? Why?

2. Consider the system as shown below, where mass  $m$  is a magnetic bar moving frictionlessly in a massless solenoid mounted on mass  $M$ . Let  $x_1$  and  $x_2$  be the absolute position of  $M$  and  $m$ , respectively. A linear force  $F$  is applied to  $m$  in the positive  $x$  direction if a positive voltage  $V$  is applied to the solenoid, and  $F = \mu V$ , where  $\mu$  is a constant. For simplicity, let  $\mu = 1$ .



- (a) (5%) Derive the governing equation and the state space realization,  $\dot{x} = Ax + BV$ , of the system.
- (b) (6%) Is the system stable? Why?
- (c) (7%) Assume zero initial condition, what is the controllable subspace of the system?
- (d) (7%) Based on Newton's law of motion, give a physical explanation of your answer to problem (c).

3. (15%) Consider the following 2<sup>nd</sup> order system

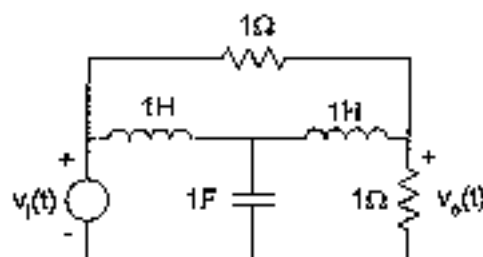


This system has a nonlinear device which saturate at 10. This system is to be excited by a sinusoidal input of the following form

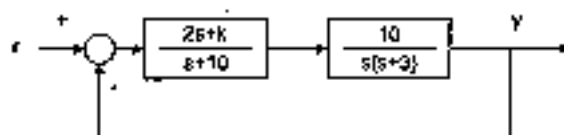
$$u(t) = A(\omega) \sin \omega t$$

Find the range of the amplitude  $A$  as a function of frequency  $\omega$  and  $k$ , so that the nonlinear device won't saturate in steady state response.

4. Determine the transfer function  $\frac{v_o(s)}{v_i(s)}$  of the following network. (12%)



5. Draw the locus of the following closed-loop system poles on the  $s$ -plan with respect to the tuning parameter  $k$  ( $k > 0$ ) and find the following: (20%)  
 (a). Asymptotes (b). Breakaway and break-in points (c). The range of  $k$  for system stability  
 (d). The value of  $k$  to yield a 0.5 damping ratio for the dominant second-order pair



6. Read the following system characteristics from the gain-phase plot of an open-loop stable system shown below. (18%)  
 (a). Gain margin (b). Phase margin (c). Closed-loop bandwidth with unity feedback  
 (d). Gain crossover frequency (e). Phase crossover frequency  
 (f). What kind of the controller can be used to increase the closed-loop bandwidth with the same phase margin?

