1. Solve the following ordinary equations:

(a) $4yy' = e^{x-y^2}$, $\quad y(1) = 2$.

(b) $(2y + e^y + 6x^2)y' + 4 + 12xy = 0$.

(c) $y'' - 4y' + 3y = 0$, $\quad y(0) = 4$, $\quad y'(0) = 0$.

(15%)

2. Use

(a) the eigenvalue-eigenvector method (10%)

(b) the Laplace transform method (10%)

to solve the following simultaneous differential equations:

$$\dot{x}_1 = -5x_1 + 2x_2, \qquad x_1(0) = 1$$
$$\dot{x}_2 = 2x_1 - 2x_2 + \sin t, \qquad x_2(0) = 0$$

3. Write the equation for the Green's theorem in the plane and prove it using $\mathbf{F} = (y^2 - 7y)\mathbf{i} + (2xy + 2x)\mathbf{j}$ and C: the circle $x^2 + y^2 = 1$. (15%)

4. (a) Let

$$f = \begin{cases} -2x, & \text{for } \; -\pi \le x \le 0, \\ 4, & \text{for } \; 0 < x \le \pi. \end{cases}$$

Determine the Fourier series, $F(x)$, and find the values of $F(-\pi)$ and $F(0)$.

(b) Let $f(x) = x^2/2$ for $-\pi \le x \le \pi$. Find the Fourier series of f and evaluate it at an approxpriately chosen value of x to sum the series

$$\sum_{n=1}^{\infty} (-1)^n (1/n^2).$$

(15%)

(背面仍有題目,請繼續作答)

5. (a) Solve the following boundary value problems:

$$\begin{array}{ll} u_t = 3u_{xx}, & 0 < x < 1, \quad t > 0, \\ u(0,t) = u(1,t) = 0, & t < 0, \\ u(x,0) = 1 - \cos(2\pi x), & 0 < x < 1. \end{array}$$

(b) Show that the partial differential equation

$$u_t = u_{xx} + 4u_x + 2u$$

can be transformed into a standard heat equation by choosing a and b appropriately in

$$u(x,t) = e^{ax+bt}v(x,t)$$

(20%)

6. What is the Cauchy-Riemann equations? Prove it. (15%)