1. The two-dimensional velocity potential of the inviscid flow field of an uniform flow over a rotating circular cylinder is

$$\phi = -V_o r\cos\theta - \frac{\chi\cos\theta}{r} + \frac{\Lambda}{2\pi}\theta$$

where r and θ are the two-dimensional polar coordinates with the origin locates at the cylinder center line as shown in the attached figure, V_o is the free stream velocity, χ is the doublet strength, and Λ is the vortex strength. The cylinder surface is at the radius of $r = \sqrt{\frac{\chi}{V_o}}$.

a. Prove that the flow field is irrotational. (8%)

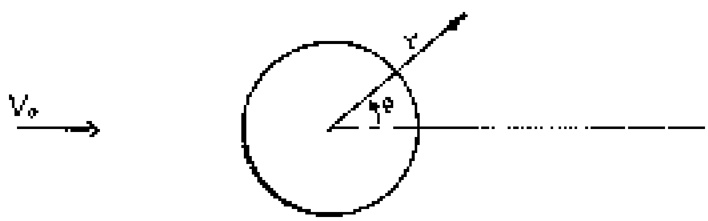
b. For a positive and negative Λ, draw a schematic diagram to indicate the corresponding effective force directions induced by the vortex and explain (12%).

Hint: For the cylindrical coordinate system the gradient and curl operator can use the following equations.

$$\nabla f = \frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{\partial f}{\partial z}\vec{e}_z$$

$$\nabla \times \vec{A} = \begin{bmatrix} \vec{e}_r & r\vec{e}_\theta & \vec{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & rA_\theta & A_z \end{bmatrix}$$

$$\vec{A} = A_r\vec{e}_r + A_\theta\vec{e}_\theta + A_z\vec{e}_z$$



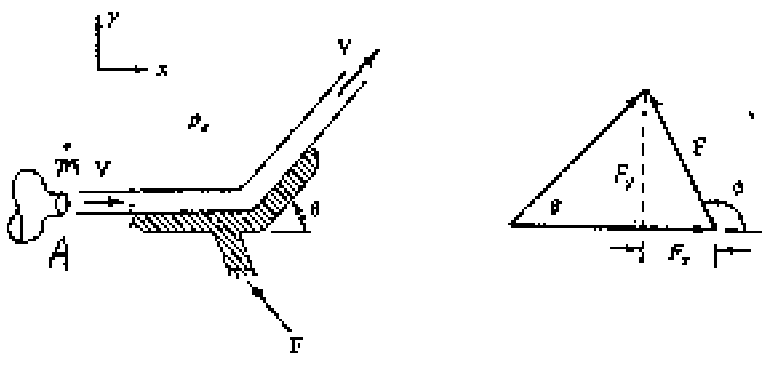
(背面仍有題目,請繼續作答)

2. The appropriate equation for fully developed flow in a circular pipe of radius R is

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}=\frac{1}{\mu}\frac{dp}{dx}$$

Find the appropriate dimensionless parameters and the dimensionless equation. The reference value for u is the centerline velocity U_0. Also find the dimensionless boundary conditions. (20%)

3. (20%)
As shown in Fig. 1, a fixed vane turns a water jet of velocity V, mass flow rate $\dot{m}$, and area A, through an angle θ without changing its velocity magnitude and mass flow rate. Assume that the flow is steady, pressure is P_a everywhere, and friction on the vane is negligible. (a) Find the components F_x and F_y of the applied vane force. (b) Find expressions for the magnitide F and the angle ϕ in terms of given parameters. (c) Plot the curves for $\frac{F}{\dot{m}V}$ and ϕ versus θ in a figure.



(a) (b)

Fig. 1.

Master's Entrance Exam., Viscous Flow, 1998

(20%) 4. Suppose a viscous flow has a velocity distribution

$$\vec{v}(x,y,z) = (3z)\vec{i} + (-5x)\vec{j} + (-2y)\vec{k}$$

where (x,y,z) are the Cartesian coordinates and $\vec{i}, \vec{j}, \vec{k}$ are unit vectors in x,y,z directions, respectively. The constant molecular viscosity is μ. A fluid element of volume dV is located at $(1,0,1)$.

(a) Verify whether this flow is incompressible or compressible.

(b) Calculate the angular velocity of this fluid element. Is this a rotational or irrotational flow?

(c) Suppose the fluid element has a surface described by

$$d\vec{s} = |ds|\left(\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}\right) = |ds|\vec{n}$$

where $\vec{n}$ is the outward unit normal vector and $|ds|$ is the magnitude of the surface area. Find the viscous force (a vector) acting on the fluid element through $d\vec{s}$ by the fluid outside the element.

5. (20%)

A sphere of specific gravity 7.8 is dropped into oil of specific gravity 0.88 and viscosity $\mu = 0.15\,Pa \cdot s$. Under the assumption of creeping flow, estimate the termial velocity of the sphere if its diameter is 0.1mm. (density of water at 20°C is equal to 998 kg/m^3).