16 % 1. Let A,B,C,D $\in C^{n\times n}$. Consider the maps

i. $X \to AX + XB$,

ii. $X \to AX + BXC$,

iii. $X \to AX + XBX$,

iv. $X \to AX + XB + CX^T$

are they linear or not, Give brief proof or counter example.

20% 2. Consider a realization $\{A,b,c\}$ with

$$A = \begin{bmatrix} -0.5 & 1 & 0 \\ -1 & -0.5 & 0 \\ 0 & 1 & 0 \end{bmatrix},\quad b = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix},\quad c' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

1. Is the system observable? If not completely observable, what quantities are unobservable?
2. In the system controllable? If not completely controllable, what quantities are uncontrollable?

14 % 3. Suppose A is a linear operator from x into x, a two dimensional vector space. Let u_1, u_2 be a basis for x and

$$Au_1 = u_1 + 2u_2$$

$$Au_2 = 3u_1 + u_2$$

What is the matrix representation of A with respect to u_1, u_2?

Let $\begin{aligned} V_1 &= u_1 + u_2, \\ V_2 &= u_1 - u_2 \end{aligned}$. What is the matrix representation of A with respect to V_1, V_2?

(4a) 5% What are the purposes of a control system?

(4b) 5% Plot a block diagram for a typical control system and explain the functions for each component.

(4c) 5% Is a human being himself a control system? Why?

(5) Given a system described by the following input-output relation:

$$\frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

(a) 10% find the gain margin in terms of ω_n and ξ.

(b) 10% find the phase margin in terms of ω_n and ξ.

(背面仍有題目,請繼續作答)

(6) 15% Consider a system described by

$$\frac{d^2y(t)}{dt^2} + a\frac{dy(t)}{dt} + by(t) = u(t),\ y(0) = 1,\ \dot{y}(0) = 0$$

where $u(t)$ is the control input, $y(t)$ is the output, and a, and b are two given constants. Design a control input $u(t)$ using the feedback of $y(t)$ and $\dot{y}(t)$ such that the output of the system will be in the form

$$y(t) = C_1e^{-\alpha t} + C_2e^{-\beta t}$$

where α and β are two given constants. Can you determine C_1 and C_2 uniquely?