1. (14 points)

Find the radius of convergence and interval of convergence of the power series:

a). $\sum_{n=0}^{\infty} \frac{1}{4n+2}(x+1)^n$,

b). $\sum_{n=0}^{\infty} (-\frac{3}{2})^n (x-3)^n$,

c). $\sum_{n=0}^{\infty} \frac{1}{n^2 3^n}(x+1)^n$.

2. (14 points)

Solve the integral equation for $y(t)$

$$y(t) = t^2 - 4\int_0^t (t-\tau)y(\tau)\,d\tau.$$

3. (16 points)

Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

a). $\det A =$?

b). $\det(A^T A) =$?

c). eigenvalue $\lambda(A) =$?

d). How many linearly independent eigen-vectors does A have?

4. (14 points)

The equation of an ellipsoid is

$$x^2 + y^2 + \frac{z^2}{9} = 1.$$

If

$$\vec{f} = x^2yz\vec{i} + xy^2z\vec{j} - 2xyz^2\vec{k},$$

evaluate the surface integral

$$\oint_S \vec{f}\cdot\hat{n}\,dA$$

where S is the ellipsoid surface and $\hat{n}$ is the outdraw normal.

(背面仍有題目,請繼續作答)

5. (14 points)

Determine the solution of the following simultaneous equations,

$$\frac{dx}{dt} = x + y, \qquad x(0) = 1,$$

$$\frac{dy}{dt} = -2x + 4y + 1, \qquad y(0) = 0.$$

6. (14 points)

Find the solution of $u(x, y)$ of the equation

$$\frac{\partial^2 u}{\partial x \partial y} - u = 0$$

with the conditions of $u(0, 1) = 1$ and $u(1, 0) = e$.

7. (14 points)

a). Find and classify all singular point(s) of the complex function

$$f(z) = \frac{e^{iaz}}{z^2 + 4}.$$

b). Find the residues of the simple pole(s).

c). Evaluate the following integral

$$\int_0^\infty \frac{\cos ax}{x^2 + 4}\, dx.$$