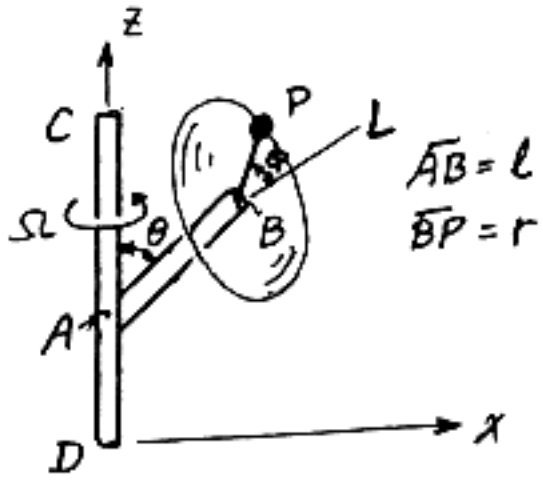
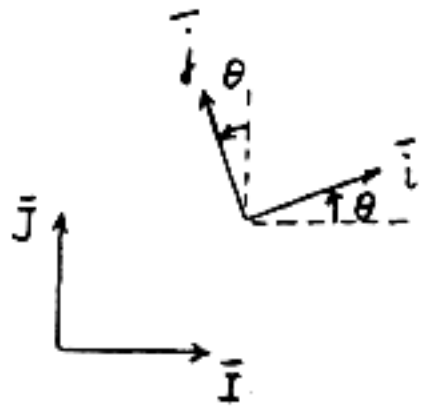
Problem 1. As shown in the following figure, struts AB and CD are fixed in the x-z plane. The entire structure rotates with angular velocity Ω about the z- axis. Particle P rotates with angular velocity $\dot{\phi}$ about AB with BP being always vertical to strut AB. Assume that BL is the base line parallel to the y- axis. For the position of the particle P as shown, determine the velocity $\mathbf{v}_P$ and accleration $\mathbf{a}_P$ of particle P. Resolve $\mathbf{v}_P$ and $\mathbf{a}_P$ in xyz components. (20%)



Problem 2. As shown in the sketch, IJ and ij are the unit vectors associated with XY and xy coordinate systems, respectively.

a. Determine ij in terms of IJ and θ (the angle between i and I). (10%)

b. Let a vector $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j}$ where x and y are given. If the vector is to be expressed as $\mathbf{r} = X\,\mathbf{I} + Y\,\mathbf{J}$, what are the components X and Y in terms of x, y and θ? (10%)



Problem 3. One solar day is the time required for an observer on the Earth to revolve once and observe the Sun at the same location. It is well known that 1 mean solar day = 24 hours.

a. If one year = 365.25 mean solar days, what is the angular velocity (radian per hour) of the Earth revolving about its own axis? (10%)

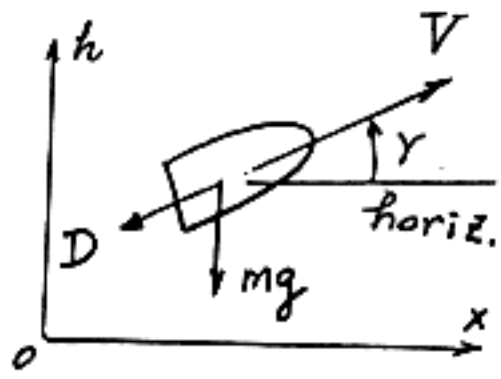
b. A "geostationary" satellite is the one which remains over one spot on the Earth at all times. Assume that the gravity

$$g = g_0 \frac{R_E^2}{(R_E + h)^2},$$

where R_E is the radius of the Earth (assumed to be a sphere), h the altitude and g_0 the gravity on the Earth surface. For a circular orbit above the equator of the Earth, what is the altitude of a geostationary satellite? (10%)

(背面仍有題目,請繼續作答)

Problem 4. A bullet with mass m is subject to the aerodynamic drag D and the gravity force mg as shown in the sketch. The velocity and the flight path angle of the bullet are V and γ, respectively. Determine $\dot{x}$, $\dot{h}$, $\dot{V}$, and $\dot{\gamma}$ (the time derivatives of the state variables) in terms of V, γ, D, m, and g. (20%)



Problem 5. Three masses at points A, B, and C with mass $1m$, $2m$, and $3m$, respectively, are joined by rigid bars of negligible mass as shown in the figure. The three masses are on a horizontal smooth surface. Being initially at rest, they are subject to a force $\mathbf{F} = F\,\mathrm{j}$ as shown. Assume that XY coordinate with unit vectors **IJ** is a fixed frame and xy coordinate with unit vectors ij is a moving frame. Determine the position vectors of points A, B, and C as functions of time. Express the position vectors in XY frame. (20%)