(15%)

1. Find the radius of convergence and interval of convergence of the power series:

   a) $\sum_{n=0}^{\infty} \frac{1}{3n+1}(x+2)^n$, b) $\sum_{n=0}^{\infty}(-\frac{2}{5})^n(x-1)^n$, c) $\sum_{n=0}^{\infty} \frac{1}{n^2 2^n}(x+1)^n$.

(20%)

2. Find a complete solution of the equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x.$$

(15%)

3. Consider the matrix $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, please find (a) its characteristic polynominal

(or characteristic equation), and (b) the corresponding eigen-values and their associated eigen-vectors.

(15%)

4. Find a unit normal vector to the curve $4x^2+9y^2 = 36$ at point $(3\sqrt{2}/2, \sqrt{2})$, by using

(a) the Cartesian coordinate (x, y) system,

(b) the polar coordinate (r, θ) system, where $r \triangleq \sqrt{x^2 + y^2}$ and $\theta \triangleq \tan^{-1}(y/x)$.

(15%)

5. 複數 z 可表成 $z = x + iy$，其中，$i = \sqrt{-1}$，若 z 的共軛複數(complex conjugate)是 z^*，請分別判定下列函數是否是解析函數(Analytic Function)？

   a). $f(z) = e^z$,

   b). $f(z) = z \cdot z^*$,

   c). $f(z) = z^3$.

(20%)

6. (a) Solve the problem of two dimensional Laplace equation (15 %)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$
$$x = 0, \quad T = 0$$
$$x = 1, \quad T = 0$$
$$y = 0, \quad T = \sin(2\pi x)$$
$$y = 1, \quad T = 0$$

(b) If the boundary conditions becomes

$$x = 0, \quad T = \sin(4\pi y)$$
$$x = 1, \quad T = 0$$
$$y = 0, \quad T = \sin(2\pi x)$$
$$y = 1, \quad T = 0.$$

and you are asked to solve the problem via the method of separation of variables. Show the procedure of how to the problem (do not solve the problem in detail). (5%)