1. (20%) As a construction engineer, you are trying to estimate the aerodynamic forces exerted on a semi-circular building. The airflow is assumed inviscid and incompressible. The stream function in polar coordinates for the air outside the building is taken to be

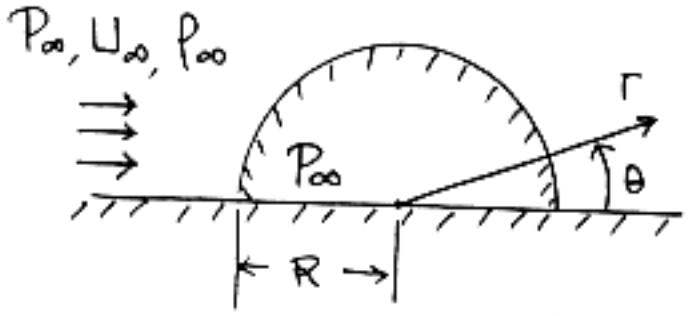
$$\psi = U_{\infty} r(\sin\theta)(1 - \frac{R^2}{r^2})$$

where U_{∞} is the incoming wind speed at far upstream and R is the radius of the building. Suppose the pressure inside the building is the same as the atmospheric pressure at far upstream, P_{∞}. The air density is ρ_{∞}.

(a) What is the lift force experienced by the building?

(b) What is the drag force experienced by the building?

Hint: $\int(\sin^3\theta)d\theta = \frac{1}{3}(\cos^3\theta) - \cos\theta$



2. (20%) Use the concept of a control volume to describe a conservation law. Namely, derive an expression for dX/dt in a control volume, where X denotes the property of a fixed mass system of fluid particles. Explain the physical meaning of each term obtained in your expression for dX/dt.

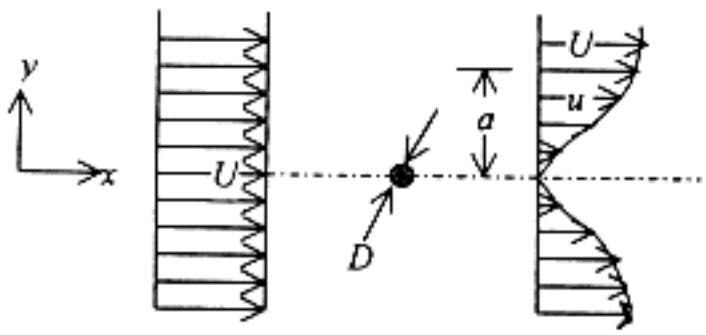
(20%) 3. What is the concept of Area Ruling in aerodynamics? What kind of the flow and what kind of aerodynamic parameter(s) in aircraft design will the area ruling be very important to be applied? Explain the reasons in detail and use example(s) and drawing(s) to help describe your statement. (20%)

(20%) 4. Consider an aircraft with wing of symmetric airfoil, the drag coefficient is $C_D=C_{D0}+kC_L^2$, where C_{D0} is the zero lift coefficient and C_L is the lift coefficient. Assume that C_{D0} and k are constant, show that the lift coefficient for maximum lift/drag ratio and the maximum lift/drag ratio for an incompressible flow are given by

$$C_{L(L/D)max} = \sqrt{(C_{D0}/k)}$$

$$(L/D) = 1/(2\sqrt{kC_{D0}})$$

(20%) 5. Experimental measurements are made in a low-speed air jet to determine the drag force on a circular cylinder. Velocity measurements at two sections, where the pressure is uniform and equal, give the results shown. Evaluate the drag force on the cylinder, per unit width.



U = 60 m/s

ρ = 1.2 kg/m^3

D = 20mm

$a = 2.2D$

$u = U\sin\left(\frac{\pi y}{2a}\right); 0 \le y \le a$

$U = U \quad ; y>a$