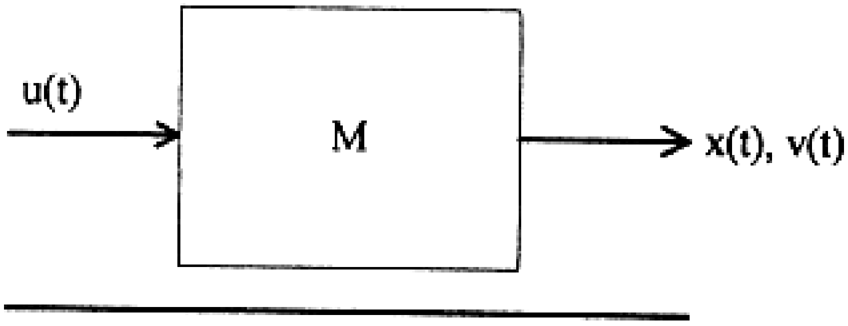
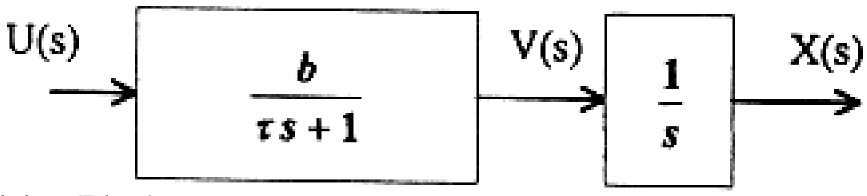
(共計五大題，每大題二十分)

1 Consider a positioning system sketched below.



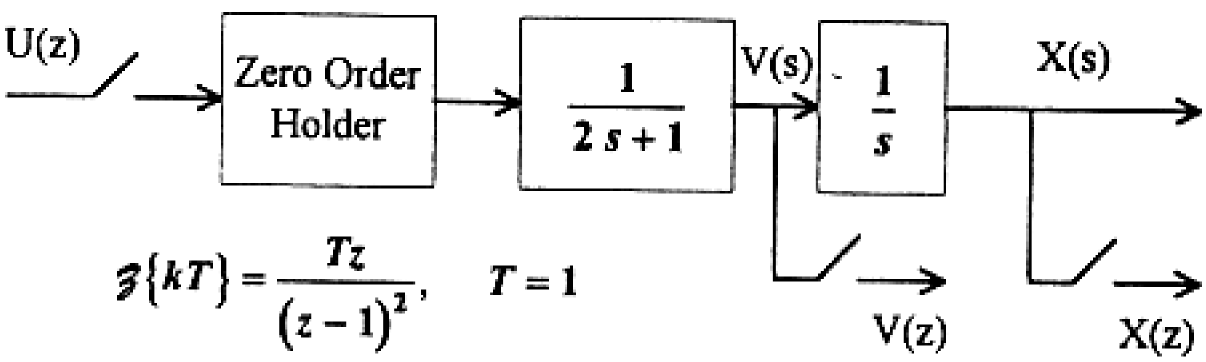
Viscous Friction =Bv
u(t): Input Force,
v(t): Velocity,
x(t): Position.

The transfer function model can be obtained as shown in the block diagram below.



(a) Find b and τ.

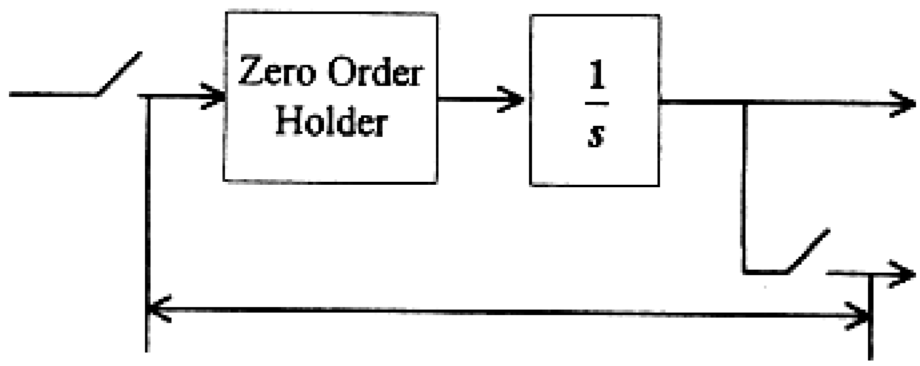
(b) The input force is held to the value $u(k)$ from $t=kT$ to $t=(k+1)T$ where k denotes the k-th time instance that the velocity and position of the mass $v(k)$ and $x(k)$ are sampled. To simplify the calculation, the constants are set as in the block diagram below.



$$\mathcal{Z}\{kT\} = \frac{Tz}{(z-1)^2}, \quad T=1$$

In Z-domain, find the z-domain relation between V(z) and X(z): i.e. $G_{vx}(z) = \frac{X(z)}{V(z)}$.

(c) Does your answer in (b) should agree with the discrete time transfer function $G(z)$ the you obtain for a pure integrator preceded by the zero order hold in the figure below? Give yes/no answer with brief but clear supporting statement.



G(z)

2 Given a system described by the following input-output relation:

$$\frac{Y(s)}{U(s)}=\frac{4}{s^2+4.5s+4}$$

(a) Find the natural frequency ω_n and damping ratio ξ for the given system.

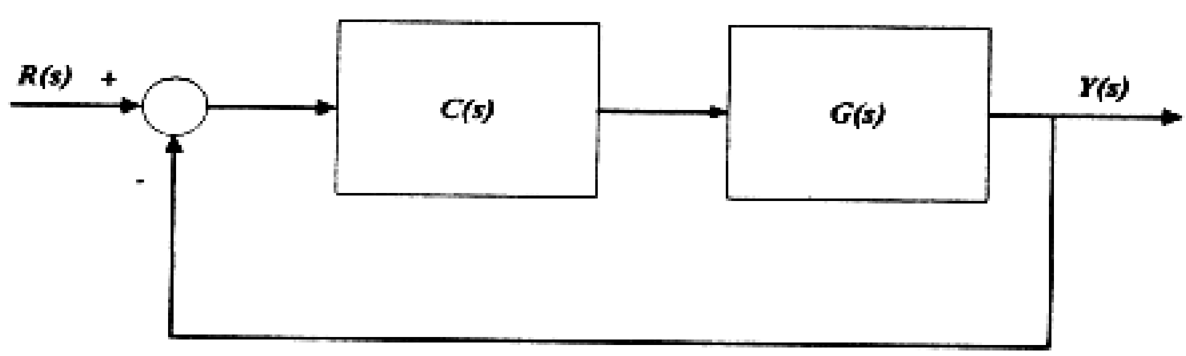
(b) Find the gain margin and phase margin for the given transfer function.

3 Consider a system described by $\frac{d^2y(t)}{dt^2}+(-3)\frac{dy(t)}{dt}+2y(t)=u(t)$ with initial conditions $y(0)=1,\ \dot{y}(t)=0$. Design a control input $u(t)=ay(t)+b\dot{y}(t)$ such that the output of the given system will be $y(t)=C_1e^{-t}+C_2e^{-2t}$. Determine $a,\ b,\ C_1,\ C_2$.

4 Consider the following system with $G(s)=\frac{30}{s^3+11s^2+10s}$.

(a) Plot the Bode plot of $G(s)$.

(b) Determine the parameters of the controller $C(s)=\frac{k(s+z)}{s+p}$ by root locus method, such that the dominant closed loop poles are located at $-2\pm 2j$.



5 Consider a standard mass-spring-damper system $m\ddot{x} + c\dot{x} + kx = u$, where m, c, k are mass, damping coefficient, and spring constant respectively, and x and u are mass position and input force. Is the condition $\begin{cases} x = x_d > 0, \\ \dot{x} = 0, \\ u = 0 \end{cases}$ stable or not? Why?