Problem 1. As shown in Fig. 1, a rod is moving in the x-y plane. Assume the distance of points A and L from the left end of the rod is a and l, respectively. Given (x_A, y_A), ω, and $\theta(0)$, i.e., the position of point A, the angular velocity of the rod, and the initial angle of the rod, respectively, as follows:

$$x_A = t + 2t$$
$$y_A = -3t - t^2$$
$$\omega = \sin 2t$$
$$\theta(0) = 0$$

Determine $\mathbf{r}_L$, $\dot{\mathbf{r}}_L$, and $\ddot{\mathbf{r}}_L$, i.e., the position vector, the velocity, and the acceleration of L, repectively.

- Express $\mathbf{r}_L$, $\dot{\mathbf{r}}_L$, and $\ddot{\mathbf{r}}_L$ in $\mathbf{i}, \mathbf{j}$ unit vectors, which are in a fixed frame. (15%)
- Express $\mathbf{r}_L$, $\dot{\mathbf{r}}_L$, and $\ddot{\mathbf{r}}_L$ in $\mathbf{e}_L, \mathbf{e}_\theta$ unit vectors, which are in a frame fixed to the moving rod. (10%)

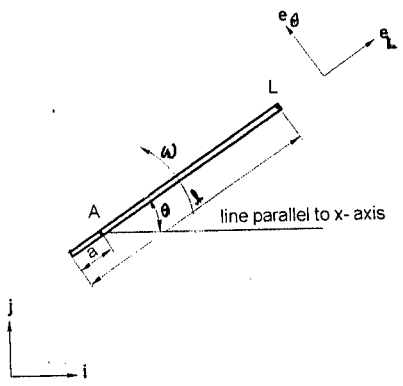

Figure 1: Figure for Problem 1.

Problem 2. Consider the plane motion of a rocket as shown in Fig. 2. Define

V = velocity (along the axis)
γ = angle between V and horizon
T = thrust
L = lift (vertical to velocity)
D = drag (parallel to velocity)
m = mass
g = gravity
t = time

- Using the symbols defined above, determine the acceleration $\dot{V}$ and the angular velocity $\dot{\gamma}$. (18%)
- Assume that T is constant, L and D are proportional to V^2, describe how you can solve the problem. (7%)

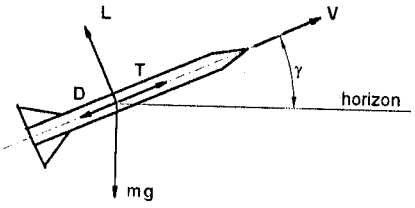

Figure 2: Figure for Problem 2.

Problem 3. Consider a cylinder rolling on a cart as shown in Fig. 3. There is no friction on the floor and the cylinder is rolling without slip on the cart. Derive the dynamic equations of motion for the system. For small θ, linearize the equations of motion. (25%)

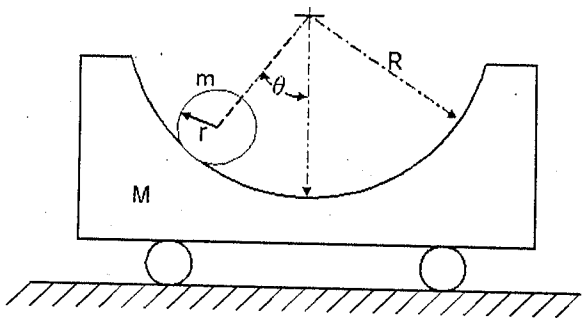

Figure 3: Figure for Problem 3.

Problem 4. As shown in Fig. 4, the motor has a mass of M and radius of gyration k about the z-axis. Motor shaft is supported by bearing at A and B, and is rotating at a constant rate of ω_s. Also, the frame rotates at the rate of ω_y about y-axis. Determine the reaction forces exerted on bearing A and B, respectively. (25%)

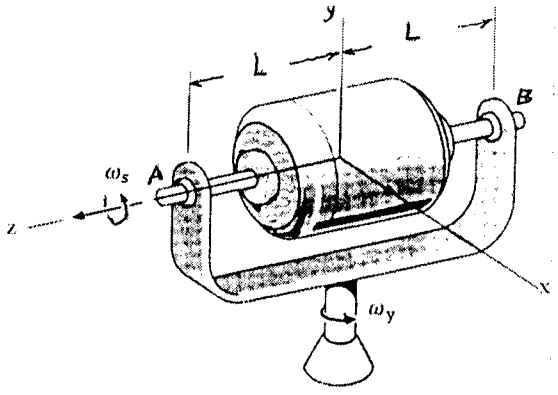

Figure 4: Figure for Problem 4.