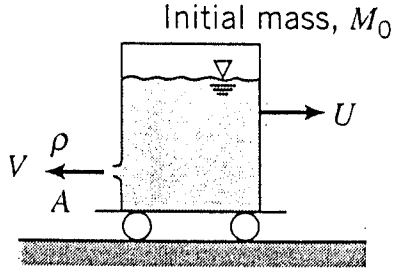
Fluid Mechanics

1. (1) Write down equations that can describe streamlines. (4%)
Let the fluid velocity be (u, v, w). Draw a picture for explaining the relation of the velocity and the streamlines. (4%)
(2) If u = x, v = 1, w = x, and one of its corresponding streamline passes through the point (1, 0, 0), find the streamline? (7%)
(3) Can a particle path be a streamline? (2%) In what situation, a particle path is a streamline? (2%) Explain the physical meaning of Df/Dt, where D/Dt denotes the substantial (or material) derivative and f(t, x, y, z) is the fluid property. (3%) Write down the detail of Df/Dt. (3%)

2. A cart is propelled by a liquid jet issuing horizontally from a tank as shown. The track is horizontal; resistance to motion may be neglected. The tank is pressurized so that the jet speed may be considered constant. Obtain a general expression for the speed of the cart as it accelerates from rest. (25%)



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3. Consider a steady velocity field $\vec{V}(x, y) = (-kx,\ ky)$ where (x,y) is the position in Cartesian system and $k>0$ is a constant. Suppose at time $t=0$ a fluid element P is located at (x_0, y_0).

(a) Find the position (x,y) of the fluid element P at time $t>0$ as a function of x_0, y_0 and t. (15%)

(b) Let us follow the fluid element P as it flows in this velocity field. What is the acceleration experienced by P as it flows? (10 %)

(25%) 4. An incompressible fluid flows in a pipe of radius, R. At the inlet, section 1, the velocity is uniform over the cross-section, with a value V_1. At section 2, where the flow is laminar and fully developed, the velocity varies with radius according to the relation

$$V = V_{max}(1 - \frac{r^2}{R^2})$$

a) Demonstrate that $\frac{V_1}{V_{max}} = \frac{1}{2}$

b) If $\bar{\tau}_w$ is the average wall shearing stress retarding the flow between sections 1 and 2, find the pressure drop $(p_1 - p_2)$ in terms of $V_1, \rho, L, R, \bar{\tau}_w$