碩士班入學考試
空氣動力學

1. A NACA0012 airfoil is moving at angle of attack α =5 degrees with the flow (air).

(1) Sketch the streamlines and pressure distributions on the upper and lower surfaces. Where is the largest pressure difference across the airfoil? Why? (5%)
(2) If the airfoil is 2 foot in chord length, the relative free stream velocity is 40 ft/sec, calculate the lift (in pound force/ft) on the airfoil (approximate) using 2D theory. The air density is 0.00238 slug/cubic ft. (15%)
(3) Sketch the lift vs angle of attack curves for 2D and 3D NACA0012 airfoils, describe the key points between them. (5%)

2. (1) Show that the lift coefficient per unit span of a cylinder with radius R for an inviscid incompressible flow is

$$C_l = -\frac{1}{2}\int_0^{2\pi} C_p \sin\theta d\theta$$

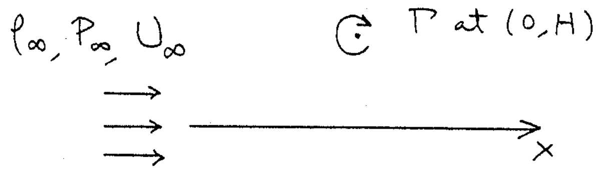
where $C_p = 1 - \frac{V^2}{V_\infty^2}$ and $\vec{V}$ is the flow velocity. (5%)

(2) Consider the flow field by superimposing a vortex with clockwise circulation (Γ) on a doublet/uniform-flow combination. What is the resultant potential of this flow? (5%) Calculate the pressure (10%) and lift coefficients (per unit span) on the cylinder. (5%)

3. In the following figure, a two-dimensional free vortex of strength Γ (clockwise) is located near an infinite plane at a distance H above the plane. In Cartesian system, the vortex is located at $(0, H)$, where the plane is taken to be the x-axis. The pressure at infinity is P_∞, the density at infinity is ρ_∞ and the velocity at infinity is U_∞ parallel to the plane. The fluid is incompressible and inviscid.

(a) Find the velocity distribution on the plane as a function of x using the image method. (15%)

(b) Find the pressure distribution on the plane as a function of x. (10%)



4. (25%) In Prandtl lifting-line theory, we assume that the circulation distribution on the finite-wing can be written as

$$\Gamma(\theta) = \Gamma(y(\theta)),\ y = \frac{b}{2}\cos\theta,\ 0 \le \theta \le \pi \quad \text{and}$$

$$\Gamma(\theta) = 2bV_\infty \sum_{n=1}^{\infty} A_n \sin n\theta \qquad (1)$$

The induced downwash is shown to be contributed by the trailing vortex system and is expressed by

$$w(y) = \frac{1}{4\pi} \int_{-b/2}^{b/2} \frac{d\Gamma(\eta)}{dy} \frac{1}{y-\eta} d\eta \qquad (2)$$

a) Explain why Eq. (1) can be expanded into Fourier Sine series only.

b) Show that the induced downwash $w(y)$ can be converted into

$$w(y) = V_\infty \sum_{n=1}^{\infty} nA_n \frac{\sin n\theta}{\sin\theta}$$

c) The total lift on the wing is

$$L = \rho V_\infty^2 b^2 \frac{\pi}{2} A_1$$

d) The induced drag is

$$D_i = \rho V_\infty^2 b^2 \frac{\pi}{2} \sum_{n=1}^{\infty} nA_n^2$$

Note that,

$$\int_0^\pi \sin m\theta \sin n\theta \, d\theta = \begin{cases} 0 & m \ne n \\ \pi/2 & m = n \end{cases}$$

$$\int_0^\pi \frac{\cos n\varphi}{\cos\varphi - \cos\theta} d\varphi = \pi \frac{\sin n\theta}{\sin\theta} \qquad n = 0, 1, 2 \cdots$$

$$\frac{1}{\pi} \int_0^\pi \frac{\sin n\varphi \sin\varphi}{\cos\varphi - \cos\theta} d\varphi = -\cos n\theta \qquad n = 0, 1, 2 \cdots$$