1. (18 points)
Consider the differential equation

$$y''(t) + y(t) = r(t)$$

where

$$r(t) = \begin{cases} 0, & \text{if } -2 \le t < 0 \\ 2, & \text{if } 0 \le t < 2 \end{cases}$$

and

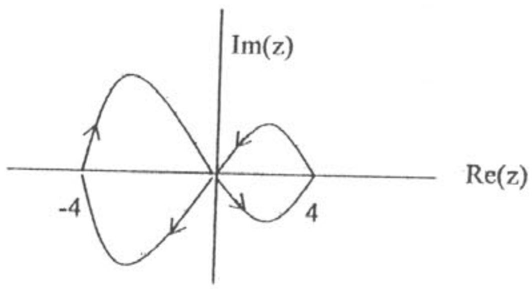
$$r(t+4) = r(t).$$

a). Find the Fourier series of $r(t)$.
b). Find the general solution of the differential equation.

2. (16 points)
Let

$$f(z) = \frac{3z}{(z+2)(z-1)^2}.$$

Evaluate $\int_C f(z)\,dz$ over the following contours:
a). $C = C_1 : |z+1| = 0.5$.
b). $C = C_2$: the boundary of the triangle with vertices 2, $2i$, and $-2i$.
c). $C = C_3$: see the figure.



z-plane

3. (16 points)
a). Find a unit vector perpendicular to the plane $4x + 2y + 4z = -7$.
b). Also, what is the shortest distance between the origin and this plane.

4. (18 points)

Solve the following ordinary differential equations:

a). $\frac{dy}{dx} + y = xe^{-x}, \quad y(0) = 1.$

b). $3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 1 = xe^{-x}, \quad y(0) = 1, y(1) = 0.$

c). $\frac{d^2y}{dx^2} + 2y = x, \quad y(0) = 0, y(\sqrt{2}) = 1.$

5. (16 points)

Find the eigenvalues and eigenvectors of the matrix

$$[A] = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}.$$

6. (16 points)

Use the Laplace transform to solve the following simultaneous equations

$$\frac{d^2y_1}{dt^2} = y_1 + y_2 + \sin(2t)$$

$$\frac{d^2y_2}{dt^2} = -4y_2 + u(t-2)e^{-2t}$$

$$y_1(0) = 1, y_2(0) = 0, \dot{y}_1(0) = \dot{y}_2(0) = 0,$$

where the unit step function $u(t-2)$ is defined as

$$u(t-2) = \begin{cases} 0, & \text{if } t \le 2, \\ 1, & \text{if } t > 2. \end{cases}$$