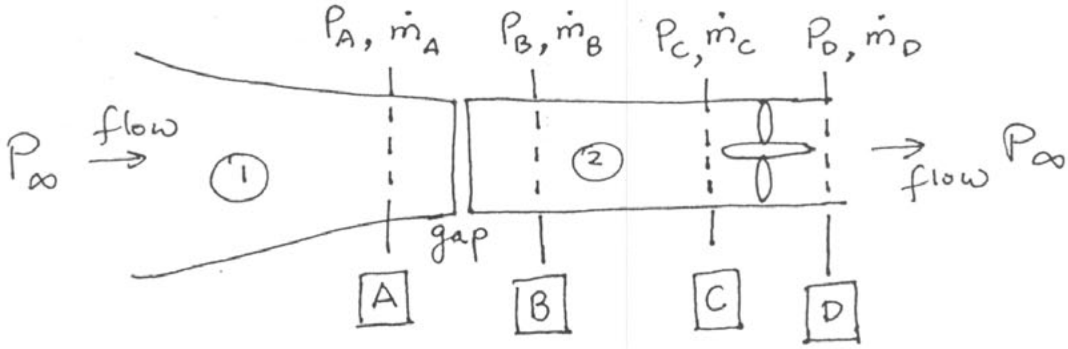
（20%）1. Consider flow driven by a fan through a duct. See the figure below.



Note That there is a gap between component ① and component ②，
see the figure.

P_∞ ：the ambient pressure

$P_{A,B,C,D}$ ：denote the state pressures at the sections A, B, C, D, respectively.

$\dot{m}_{A,B,C,D}$ ：denote the mass flow rates at the sections A, B, C, D, respectively.

Choose your answers from the following questions.

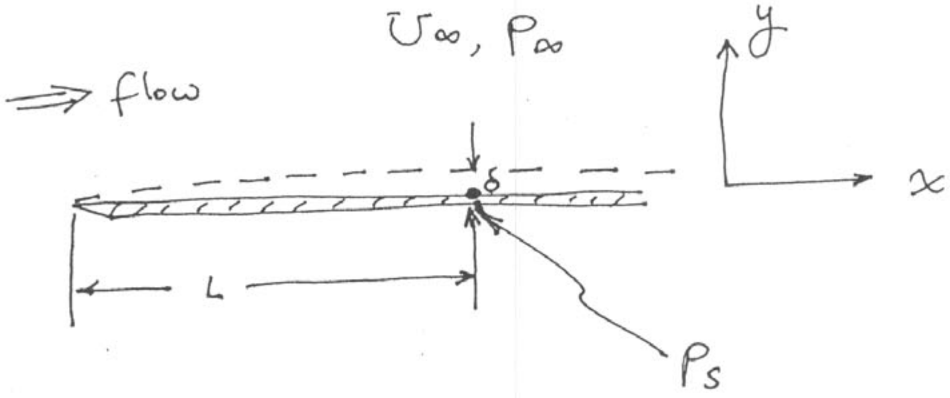
（i） （a） $\dot{m}_A > \dot{m}_B$ （b） $\dot{m}_A = \dot{m}_B$ （c） $\dot{m}_A < \dot{m}_B$

（ii） （a） $\dot{m}_C > \dot{m}_D$ （b） $\dot{m}_C = \dot{m}_D$ （c） $\dot{m}_C < \dot{m}_D$

（iii） （a） $P_A > P_\infty$ （b） $P_A = P_\infty$ （c） $P_A < P_\infty$

（iv） （a） $P_C > P_D$ （b） $P_C = P_D$ （c） $P_C < P_D$

(20%) 2. Consider a boundary layer developed on a flat plate.
See a schematic drawing below.



δ : boundary-layer thickness

L : distance from the leading-edge of the plate

U_∞ : freestream velocity

P_∞ : freestream pressure

P_s : static pressure measured on the wall

It is a common practice that the static pressure of the freestream can be obtained as the static pressure measured on the wall. Namely, $P_s = P_\infty$.
Give your explanation for this common practice.

Hint : 1. Write down the momentum equations of the x and y components.

2. Employ the boundary-layer assumption,

$\delta << L$

and check the momentum equations with the dimensional analysis

3. A steady two dimensional <u>inviscid</u> velocity field is given as

$$u(x,y) = -\cos(\pi x)\sin(\pi y),$$

$$v(x,y) = \sin(\pi x)\cos(\pi y)$$

where $u(x,y)$ and $v(x,y)$ are the x and y Cartesian components of the velocity vector.

(a) Verify whether or not this is an incompressible flow.

(b) Verify whether or not this is an irrotational flow.

(c) Verify whether or not the shear stress τ_{xy} is zero.

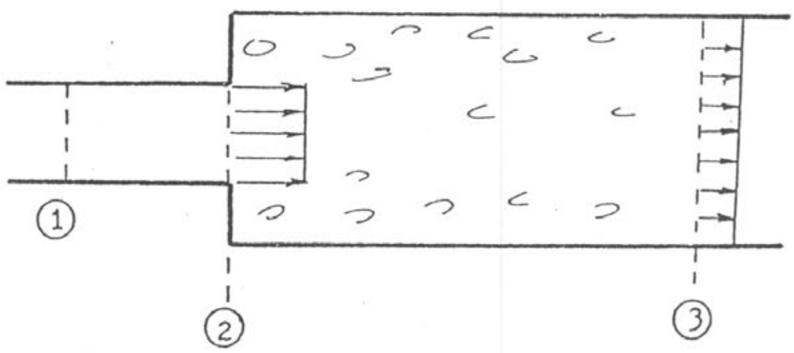
(d) It is known that the derivative of pressure with respective to y has the form

$$\frac{\partial p(x,y)}{\partial y} = C\sin(\pi y)\cos(\pi y)$$

where C is a constant. Derive the value of C from the momentum equation

20% 4. An incompressible flow undergoes a sudden expansion from a small to a large cross-sectional area. Assume that the jet profile issuing into the large area is uniform and that between section 2 and 3 the flow mixes out so that it is once again uniform. Neglect wall shear stresses. The principal assumption will be that the pressure along the entire section 2 - including the backward facing wall - is uniform. (Note: if the flow resembled a potential flow, this assumption would be invalid.)

Compute the static pressure coefficient, $(p_2 - p_1)/(\rho V_1^2/2)$ and the stagnation loss coefficient $(p_{o2} - p_{o1})/(\rho V_1^2/2)$ as a function of A_1 and A_2.



20%

5.

For an incompressible two-dimensional flow field, the velocity component in the y direction is given by the equation

$$v = x^2 + 2xy$$

a) Determine the velocity component in the x direction so that the continuity equation is satisfied.

b) Use the Navier-Stokes equations to determine an expression for the pressure gradient in the x direction.