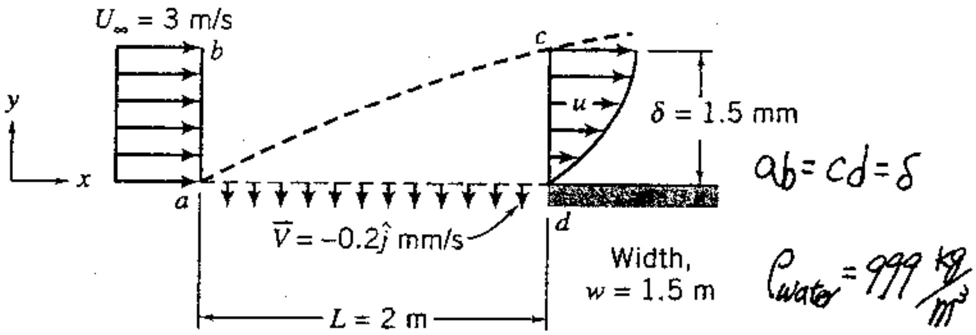
1) 20%

Water flows steadily past a porous flat plate. Constant suction is applied along the porous section. The velocity profile at section *cd* is

$$\frac{u}{U_\infty} = 3\left[\frac{y}{\delta}\right] - 2\left[\frac{y}{\delta}\right]^{1.5}$$

Evaluate the mass flow rate across section *bc*.



2) 20%

Derive the basic equation of fluid statics and apply it to obtain Archimedes' principle for a submerged object. The Archimedes' principle is that the net vertical pressure force, or buoyancy, on the object equals to the force of gravity on the liquid displaced by the object.

3) 20%

Consider the two-dimensional flow field defined by the following velocity components:

$$u = x(1+t), v = y$$

for this flow field, find the equation and draw the line of:

(a) The streamline through the point (1,2) at $t = 0$,

(b) The pathline for a particle released at the point (1,2) at $t = 0$,

(c) The streakline at $t = 0$ which passes through the point (1,2).

4) 20% (a) Assuming that the flow field is <u>irrotational</u>, please derive the <u>unsteady</u> and <u>compressible</u> governing equation for mass conservation in terms of density ρ and velocity potential ϕ. Please use $\vec{v}$ as the velocity vector, $\vec{\nabla}$ as the gradient operator and ∇^2 as the Laplacian operator. (5%)

(b) Starting from the solution of (a) and further assume that the flow is <u>steady</u> and <u>incompressible</u>, please derive the governing equation for mass conservation in terms of velocity potential ϕ. (5%)

(c) Consider a steady, irrotational, incompressible and inviscid flow field without body forces. The free stream pressure is P_∞ and the free stream density is ρ_∞. The free stream velocity is $U_\infty \vec{i}$ where $\vec{i}$ is the Cartesian unit vector along x-axis. We assume that the local flow field is a small perturbation of the free stream conditions. That is, the local velocity vector $\vec{v}$ can be written as $\vec{v} = (U_\infty + u')\vec{i} + v'\vec{j} + w'\vec{k}$ where $\vec{j}$ and $\vec{k}$ are Cartesian unit vectors along y- and z-axis, respectively; u', v' and w' are small perturbation velocities such that $|u'| << |U_\infty|$, $|v'| << |U_\infty|$ and $|w'| << |U_\infty|$. We further assume that the velocity potential ϕ can be written as $\phi = \phi_\infty + \phi'$ where ϕ_∞ is the potential for free stream velocity and ϕ' is the perturbation potential for the perturbation velocities. Please write down the Bernoulli equation relating the local flow conditions to the free stream conditions and then, derive an expression for pressure coefficient C_P in terms of U_∞ and the perturbation potential ϕ'. (10%)

5) 20%

Compute the variation of momentum thickness θ, displacement thickness δ*, and the skin friction coefficient C_f over a flat plate at zero angle of attack, as a function of x, the viscosity ν, and the freestream velocity u_e. Use Von Karman integral momentum equation. Assume the following velocity profile:

$$\frac{u}{u_e} = A + B\frac{y}{\delta} + C\left(\frac{y}{\delta}\right)^2$$

Note: von Karman integral momentum equation

$$\frac{d}{dx}\left(\theta u_e^2\right) + u_e \frac{du_e}{dx}\delta^* = \frac{\tau_w}{\rho}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_e^2}$$