25% 1. Consider the following set of system differential equations.

$$\dot{\omega}(t) + a\omega(t) + b\theta(t) - cx(t) = \tau(t),$$
$$\dot{v}(t) + dv(t) + ex(t) - f\theta(t) = 0$$
$$\dot{\theta}(t) = \omega(t), \qquad \dot{x}(t) = v(t)$$

where a, b, c, d, e, and f are constant parameters.

a. Express the equations in first-order state-space form with $\tau(t)$ as input and $x(t)$ and $v(t)$ as output.

b. Write the PD-feedback control form with constant x_d as desired input and $\tau(t)$ as output.

c. Draw a block diagram to represent the closed-loop system.

25% 2. The open-loop transfer function of a negative unity feedback system is given as

$$\frac{K(4s+5)}{s(s-2)}$$

a. Find the range of K for the system to be stable.

b. Derive the time response of the system with zero initial condition and unit-step input for $K = 1$.

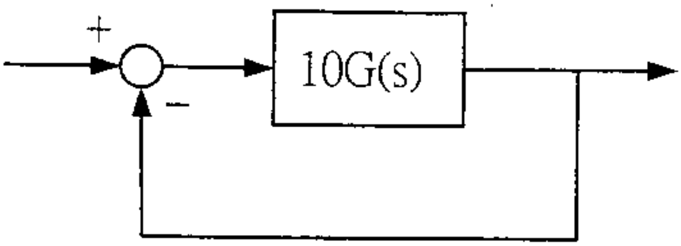
c. Evaluate the peak time and final value of the system $K = 1$.

3.

(a). (20%) Plot the Bode plot and the Nyquist plot of the following system

$$G(s) = \frac{(s+20)}{(s+2)(s-2)}$$

(b). (10%) Determine the stability of the feedback system shown below by Nyquist stability criterion.



4. (20%)

Consider the case of rabbits and foxes in Australia. The number of rabbits is x_1 with foxes present on the continent can be described as

$$\dot{x}_1 = kx_1 - ax_2$$

where x_2 is the number of foxes. Now if the foxes must have rabbits to exist, we have

$$\dot{x}_2 = -hx_2 + bx_1$$

Determine the requirements on a, b, h and k for a stable system.