Problem 1. A particle with mass m and a constant speed v is moving along a circular orbit with radius r. Derive the acceleration of this particle in terms of v and r. (20%)

Problem 2. As shown in Fig. 1, consider a point mass above the Earth of which the radius is $R_E = 6378$ km. The initial velocity of the mass on the Earth surface is $V_0 = 4000$ km/hr vertically upward. Assume that the gravity acceleration is $g = \dfrac{g_0 R_E^2}{(R_E + h)^2}$ where $g_0 = 9.8$ m/sec^2 is constant and h is the altitude of the mass. Determine the velocity V of the mass as a function of the altitude h. What is the maximum altitude the mass can reach? Compare the answer with the altitude the mass can reach if $g = g_0$. (20%)

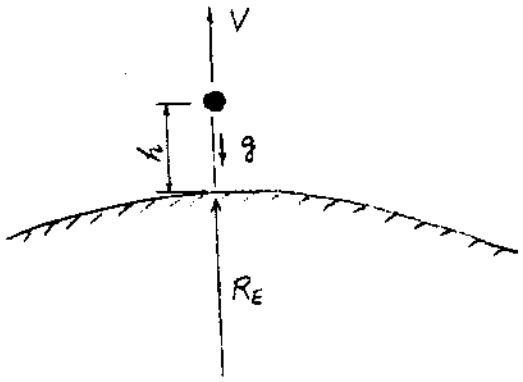

Figure 1: Schematic diagram for Problem 2.

Problem 3. A cylinder of radius R and mass m on a slope of angle ϕ is connected at the center of the cylinder by a spring with spring constant K as shown in figure 2. Consider the motion that cylinder rolls without slip.

a. Write the differential equation of motion for the system when torque T is applied to the cylinder.

b. From equilibrium position, find the angle θ that the cylinder travels when constant torque T is applied to rolls up the slope.

(20%)

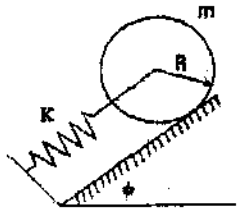

Figure 2: Schematic diagram for Problem 3.

Problem 4. Consider in Fig. 3, the ijk frame is obtained by first rotating an angle ψ about $\mathbf{K}$ to i'j'k' as shown in figure part (a) and then rotating an angle γ about j' as shown in figure part (b). If a vector is represented by $\mathbf{V} = V_x\,\mathbf{i} + V_y\,\mathbf{j} + V_z\,\mathbf{k}$, what are its components in **IJK** frame? (20%)

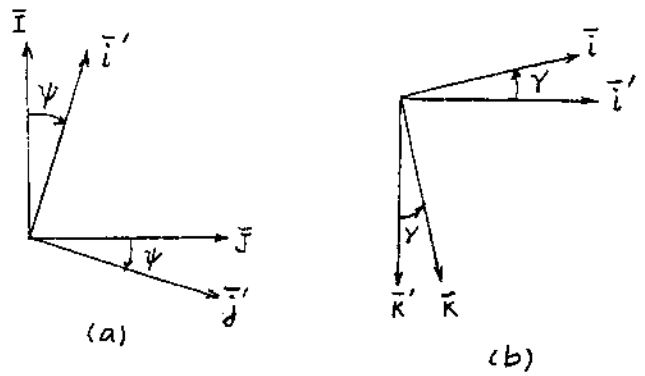

Figure 3: Schematic diagram for Problem 4.

Problem 5. In Fig. 4, the rigid body is formed with three uniform rods, each of which the length is L and the mass is $m/3$. These three rods are welted at point O, which is the origin of the attached frame ijk as shown in the figure. The body is initially static at $t = 0$. An impulse force of which the magnitude and direction can be represented with $\mathbf{F} = 2\,\mathbf{i} + \mathbf{j} + \mathbf{k}$ is acted at point O at $t = 0$. Without considering the gravity, determine the velocity and the angular velocity of this rigid body. (20%)

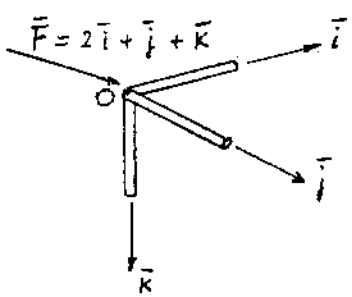

Figure 4: Schematic diagram for Problem 5.