1. (20%)

Solve the following differential equations for $y(x)$.

a) $y' - y = xy^2$, $y(0) = 0.5$

b) $y'' - y = 4xe^x$, $y(0) = 1$, $y'(0) = 0$

2. (20%)

a) Evaluate the integral

$$\int_C \frac{ie^z}{(z-2+i)^2}dz$$

where C is the counter clockwise circle $|z| = 2$.

b) Consider the mapping, $w = 1/z$. Describe and sketch

i) the image of circles which do not pass through the origin in the z plane.

ii) the image of circles which pass through the origin of the z plane.

3. (20%)

Find the radius of convergence and interval of convergence of the power series:

a) $\sum_{n=0}^{\infty} \frac{1}{n+1}(x+1)^n$,

b) $\sum_{n=0}^{\infty}(-\frac{1}{3})^n(x-1)^n$,

c) $\sum_{n=0}^{\infty} \frac{1}{n3^n}(x+1)^n$.

（背面仍有題目,請繼續作答）

4. (20%)Consider a vertical system of masses and springs. The notations in the figure are:

k_{01}, k_{12}, k_{23}, and k_{34} : spring constants

m_i, i=1,2, and 3: masses

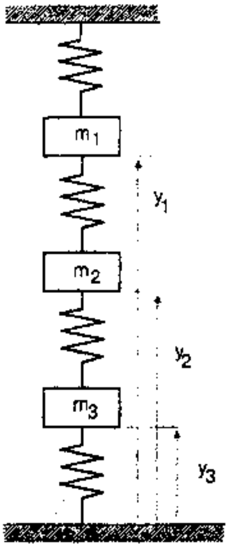
y_i: displacement of mass i from the static position

Assuming there are no frictions, the differential equations for displacements of the masses are given by

$$m_1 \frac{d^2}{dt^2} y_1(t) = -(k_{01} + k_{12}) y_1 + k_{12} y_2$$

$$m_2 \frac{d^2}{dt^2} y_2(t) = k_{12} y_1 - (k_{12} + k_{23}) y_2 + k_{23} y_3$$

$$m_3 \frac{d^2}{dt^2} y_3(t) = k_{23} y_2 - (k_{23} + k_{34}) y_3$$



Derive the eigenvalue problem associated with a harmonic oscillation. Assume that $m_1 = m_2 = m_3 = m$ =constant.

5. (20%)

Use the method of the Fourier sine function expansion to solve the problem:

PDE $\quad u_t = u_{xx} + \sin(3\pi x) \qquad 0 < x < 1$

BCs $\quad \begin{cases} u(0,t) = 0 \\ u(1,t) = 0 \end{cases} \qquad 0 < t < \infty$

IC $\quad u(x,0) = \sin(\pi x) \qquad 0 \le x \le 1$.