Problem 1. As shown in Fig. p1, a weightless rigid rod is subject to a moment, M_0, and three forces, L_1, L_2, and W, of which the distances from the left end of the rod are x_1, x_2, and x_W, respectively. The rod is in static equilibrium.

(a). If M_0, W, x_1, x_2, and x_W are given, determine L_1 and L_2. (10%)

(b). In the last case, if $M_0 = -0.5$, $W = 1$, $x_1 = 4$, $x_2 = 10$, and $x_W = 4.5$, determine L_1 and L_2. (5%)

(c). Now if x_2 is chosen to minimize the function, $D \triangleq L_1^2 + 7L_2^2$ while the other parameters remain the same as those given above, determine L_1, L_2 and x_2. How are you sure that your solutions will minimize but not maximize D? (10%)

Problem 2. For convenience of describing a kinematic problem, it is very often to adopt some multiple coordinate systems. As shown in Fig. p2, OXY is a fixed frame of which the unit vectors associated to the X- and Y- axes are $\mathbf{I}$ and $\mathbf{J}$, respectively. On the other hand, oxy is a moving frame of which the unit vectors associated to the x- and y- axes are $\mathbf{i}$ and $\mathbf{j}$, respectively. The position vector of point o relative to point O is $\mathbf{R} = X\,\mathbf{I} + Y\,\mathbf{J}$ and the velocity of point o relative to the fixed frame is $\mathbf{V} = V_x\,\mathbf{i} + V_y\,\mathbf{j}$. The angle between X- and x- axes is $\psi(t) = \Omega t$ where Ω is the angular velocity of the frame oxy relative to the frame OXY and t is the time.

(a). Given $V_x = 1$, $V_y = 2$, $\dot{V}_x = -0.1$, $\dot{V}_y = 0.3$, $\Omega = 0.5$, and $t = 3$, represent the velocity of point o in the fixed coordinate system. (10%)

(b). Use the above data to determine the absolute acceleration of point o (i.e., acceleration relative to the fixed frame) of which the components should be in the moving frame, i.e., represent the absolute accelaration as $\mathbf{a} = a_x\,\mathbf{i} + b_x\,\mathbf{j}$. (7%)

(c). Conversely, if $a_x(t) = 2$, $a_y(t) = -1$, $\Omega(t) = 0.5$, $\psi(0) = 0$, $V_x(0) = 0$, $V_y(0) = 0$, $X(0) = 0$, and $Y(0) = 0$, determine $V_x(t)$, $V_y(t)$, $\dot{X}(t)$, $\dot{Y}(t)$, $X(t)$, and $Y(t)$. (8%)

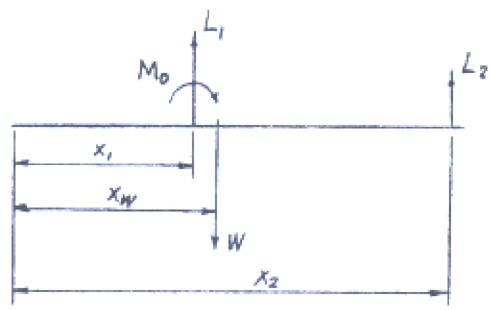

Figure p1: Schematic diagram for Problem 1.

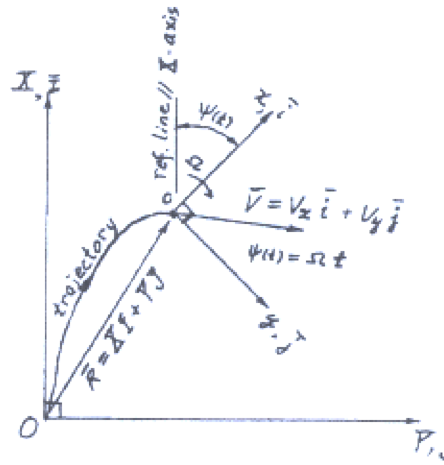

Figure p2: Schematic diagram for Problem 2.

Problem 3. A uniform slender bar of length 1 m and mass 1 kg is moving on a smooth horizontal surface. If the velocities at ends A and B are, respectively, 1 m/sec and 2m/sec in the positive x-direction when a horizontal force F = 10N is applied at end A. Determine the accelerations of ends A and B during application of force F.

(25%)

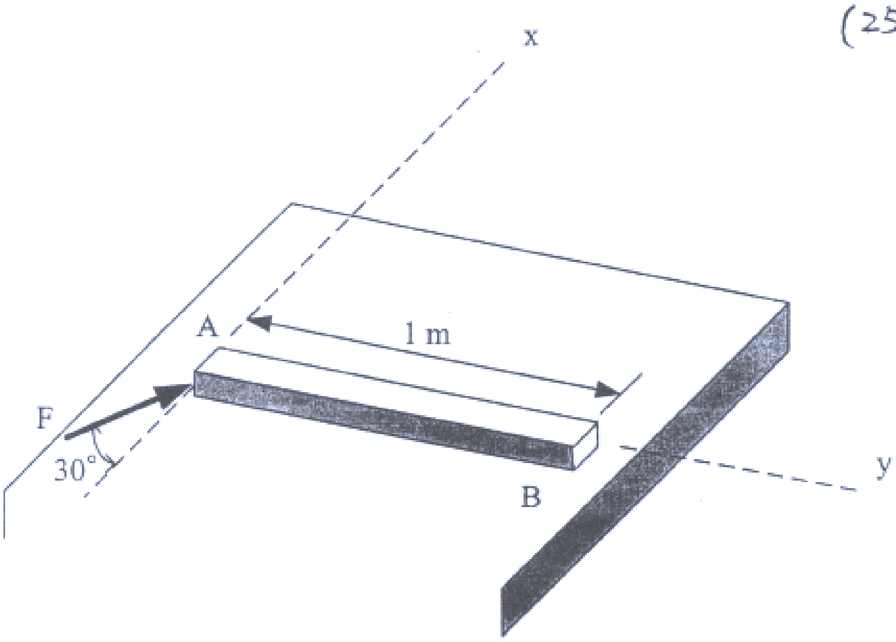

Fig. P. 3

Problem 4. The solid circular cylinder is released from rest on the 60° incline. Calculate the angular velocity ω of the cylinder and the linear velocity v of its center G after it has moved 3 m down the incline. The coefficient of friction is f = 0.3.

(25%)

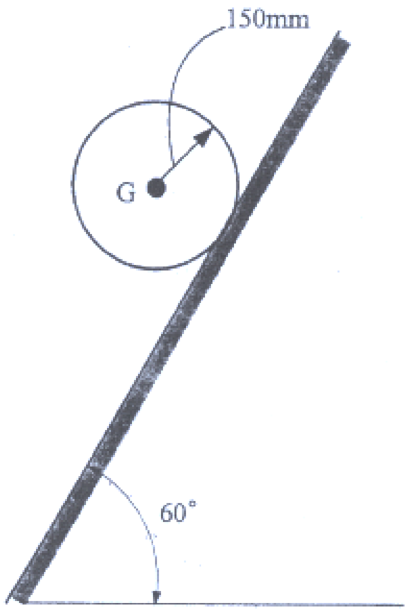

Fig. P. 4