

乙,丙,丁,戊

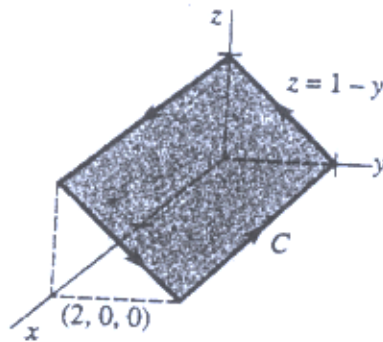
1. (20%)

(a). Solve the equation $(x+3y)dx - (x-y)dy = 0$ with $y(0) = 1$,

$$\ln(\sqrt{x+y})^{-1} = ?$$

(b). Solve the equation $y'''(x) - y''(x) - y'(x) + y(x) = e^x + e^{-x}$ for $y(x)$.

2. (20%)

Let $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$. Assume C is the boundary of the plane $z = 1 - y$ shown in the figure.(a). $\nabla \times \vec{F} = ?$ (b). Evaluate $\oint_C \vec{F} \cdot d\vec{R}$, where $d\vec{R} = dx\vec{i} + dy\vec{j} + dz\vec{k}$ is the differential displacement along C .

(背面仍有題目,請繼續作答)

3. (20%)

Consider the complex functions

a). $f(z) = \frac{1}{z^4 - z^5},$

b). $f(z) = \frac{z+1}{z^4 - 2z^3}.$

Integrate $f(z)$ **clockwise** around the circle $C: |z| = 1/2.$

4. (20%)

Use the method of Fourier sine series to solve the problem:

$$u_{tt} = 4u_{xx} \quad 0 < x < 3, t > 0$$

$$u(0, t) = u(3, t) = 0, \quad t > 0$$

$$u(x, 0) = 0, \quad 0 < x < 3$$

$$u_t(x, 0) = x(3-x), \quad 0 < x < 3$$

5. (20%)

Let $x = s + t^2$ and $y = s^3 + t$

a) Compute $\frac{\partial x}{\partial s}$ and $\frac{\partial y}{\partial t},$

b) Compute $\frac{\partial s}{\partial x}$ and $\frac{\partial t}{\partial x},$

c) Let $u = xy,$ compute $\frac{\partial u}{\partial s},$

d) Let $v = s + t,$ compute $\frac{\partial v}{\partial x}.$