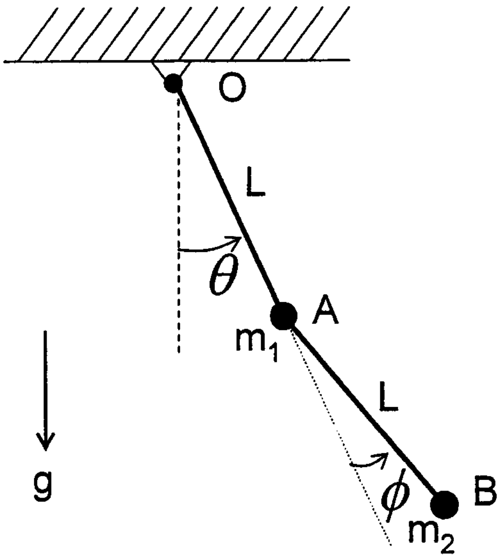
1. (25%) A double pendulum consists of two massless rods of length L, two particles of mass m_1 and m_2, and frictionless joints at points O and A. Gravity acts downward.

   a. Find the equations of motion of the system using Lagrange's equations with coordinates θ and ϕ as shown. Designate reference frames where necessary and use good notation.

   b. What forces in the system were you able to ignore by using Lagrange's method? How would they have affected an analysis based on Newton's Second Law?



2. (25%) A box, of mass m, is riding on a railroad flatcar, Fig. 2. The flatcar moves at a constant velocity of $3\,ft/\sec$ for a long time until suddenly it strikes an earth embankment and stops abruptly. What is the velocity, $u(0^+)$, of the box justA after the car hits the embankment? Assume any sort of friction between box and flatcar.

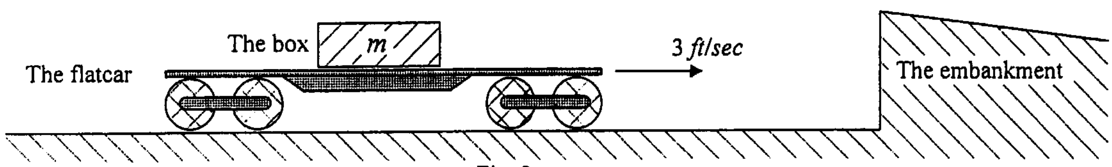
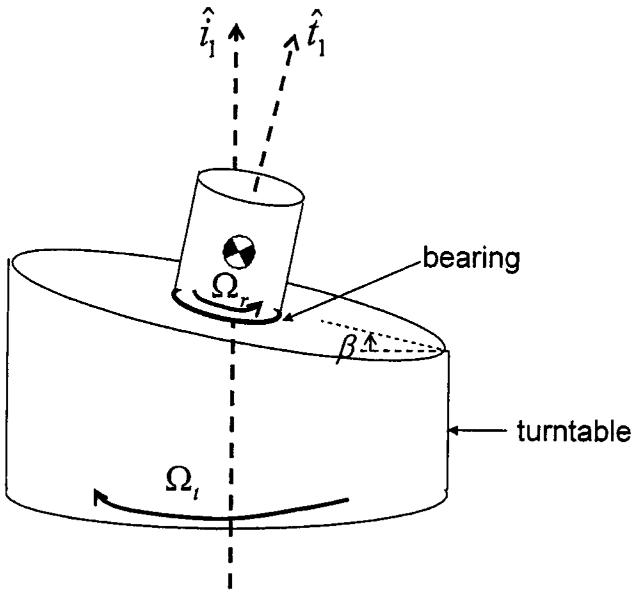


Fig. 2

3. (25%) A cylindrical turntable with its top face cut at an angle β turns with an angular velocity of ${}^{i}\vec{\omega}^{t} = -\Omega_t \hat{i}_1$ with the *i*-frame inertially fixed. On the top of the turntable is a cylindrical rotor spinning at angular velocity ${}^{t}\vec{\omega}^{r} = \Omega_r \hat{t}_1$ relative to the turntable. The rotor has moments of inertial I_A about its symmetry axis and I_T about any transverse axis passing through its center of mass. No gravitational forces are present.

   a. Specify any reference frames and coordinate bases you find convenient, express the moment the bearing exerts on the rotor about the rotor's center of mass.

   b. What relationship between Ω_t and Ω_r is needed to make this moment zero?



4. (25%) A physical system is represented by the model shown in Fig.4: a mass *m* rides on a carriage. The motion of the carriage is known, and is sinusoidal, $x(t) = x_m \cos\omega_* t$. Between the mass and the carriage is a viscous film, and thus the mass is coerced into following the carriage motion by a viscous force proportional to the relative velocity between mass and carriage, represented by damping coefficient b . Find the resulting motion $y(t)$ of the mass.

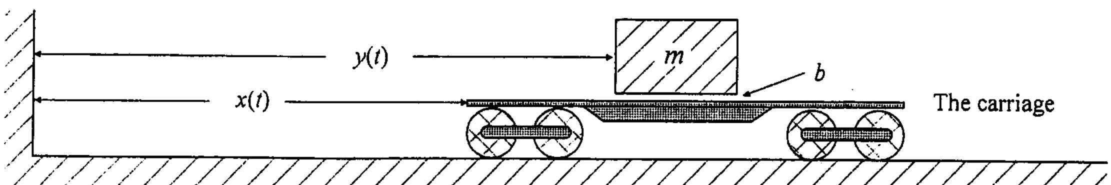


Fig. 4