1. (a) Show that the differential form

$$2\sin 2x \sinh y \, dx - \cos 2x \cosh y \, dy$$

is exact. (5%)

(b) Solve the differential equation (5%)

$$2\sin 2x \sinh y \, dx - \cos 2x \cosh y \, dy = 0.$$

(c) Solve the initial value problem. (10%)

$$y'' - y = 2\cos x, \; y(0) = 0, \; y'(0) = 3$$

2. (a) Calculate the integral (10%)

$$\int_C \vec{F}(\vec{r}) \cdot d\vec{r}, \; \vec{F} = [2z, \; 1, \; -y], \; C : \vec{r} = [\cos t, \; \sin t, \; 2t] \; from$$

$(0, \; 0, \; 0) \; to \; (1, \; 0, \; 4\pi)$.

(b) Evaluate the integral (10%)

$$\int_{(\pi/2, \, -\pi)}^{(\pi/4, \, 0)} (\cos x \cos 2y dx - 2\sin x \sin 2y dy).$$

(20pt) 3. Solve $y''' - y' = \sin t$ by Laplace transform, given $y(0) = 2$, $y'(0) = 0$, and $y''(0) = 1$. Note that $L\left\{\frac{\sin \omega t}{\omega}\right\} = \frac{1}{s^2 + \omega^2}$ and $L\{\cos \omega t\} = \frac{s}{s^2 + \omega^2}$.

4. Consider the matrix $A=\begin{bmatrix}1 & -1 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & -1 & 1\end{bmatrix}$.
(20pt)

(a). The determinant $\det(-A^{10})=?$
(b). The eigenvalues $\lambda(A^{10})=?$
(c). The rank $rank(A)=?$
(d). How many linearly independent eigenvectors does matrix A have ?

5. Let $\vec{F}=z\vec{i}+x\vec{j}+y\vec{k}$. Assume that C is the trace (邊緣) of the cylinder $x^2+y^2=1$ in the plane $y+z=2$. Orient C counterclockwise as viewed as from above. See the following figure.
(20 pt)

(a). Calculate the surface integral $I_1=\iint_S(\nabla\times\vec{F})\bullet\vec{n}dS$ directly, where $\vec{n}$ is the outward unit normal of the surface S enclosed by C, and dS is the differential area element of the surface S. (Note that you are prohibited to evaluate I_1 by using the result in (b).)

(b). Calculate the line integral $I_2=\oint_C \vec{F}\bullet d\vec{R}$ directly.
where $d\vec{R}=dx\vec{i}+dy\vec{j}+dz\vec{k}$ is the differential displacement along C.
(Note that you are prohibited to evaluate I_2 by using the result in (a).)