(1) 20%

(a.) What is the definition of a material which can be named as a fluid?
Describe its major physical characteristics. (3%)

(b.) What are the definitions of an incompressible flow and an irrotational flow?
Describe their major physical characteristics. (5%)

(c.) What is the definition of a streamline in a flow field?
Describe its major physical characteristics. (4%)

(d.) What is the definition of a streak line in a flow field?
Describe its major physical characteristics. (4%)

(e.) What is the definition of Reynolds number in a flow field?
Describe its major physical characteristics. (4%)

(2) 20%

Consider the steady flow field between two infinite horizontal walls in the figure below. The wall at $y=0$ moves at a steady velocity $U_0\hat{i}$, and the wall at $y=a$ moves at a steady velocity $2U_0\hat{i}$, where $\hat{i}$ is the unit vector in $x$-direction (horizontal).

(a) Assume that the flow velocity is in $x$-direction only. That is, $\vec{v} = U(y)\hat{i} + 0\hat{j}$, where $\hat{j}$ is the unit vector in $y$-direction (vertical). Verify whether this flow field is incompressible or compressible.

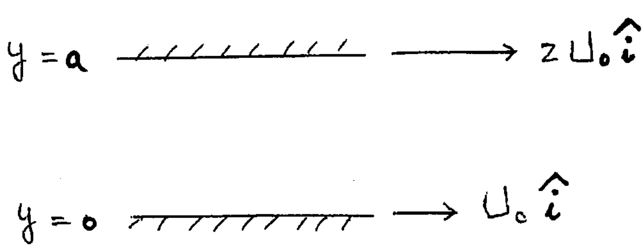
(b) The two-dimensional momentum equation in vector notation is

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right) = -\vec{\nabla}P + \mu\nabla^2\vec{v}$$

where $\rho$ is density, $P$ is pressure, $\mu$ is viscosity, $\vec{\nabla}$ is the gradient operator, and $\nabla^2$ is the Laplacian operator. For the flow field in (a), simplify the above equation to obtain an equation for $U(y)$.

(c) Assume that the pressure is constant everywhere. Solve for $U(y)$ with appropriate boundary conditions.

(d) What is the viscous shear stress experienced by the upper wall at $y=a$? Is this stress in $+x$ or $-x$ direction?



(3) 20%

In two-dimensional incompressible, inviscid flow field,

a) Please write down the equation of continuity and the definition of stream function,

b) The definitions of vorticity and irrotational flow,

c) For an irrotational flow, give the definition of velocity potential and the equation that it satisfies,

d) Write down two different types of irrotational flow and their solutions.

(4) 20%

For the flat-plate steady incompressible laminar boundary-layer flow， coefficients $a_0$, $a_1$, and $a_2$ of the assumed velocity profile $\frac{u}{U} = a_0 + a_1 \frac{y}{\delta} + a_2 (\frac{y}{\delta})^2$ are decided by the three physical conditions at the wall and boundary - layer edge. By using this velocity profile and integral momentum relation ( $C_f = 2\frac{d\theta}{dx}$ ), compute

$(a)\frac{\theta\sqrt{\mathrm{Re}_x}}{x}$ ; $(b)\frac{\delta^*\sqrt{\mathrm{Re}_x}}{x}$ ; $(c)\frac{\delta\sqrt{\mathrm{Re}_x}}{x}$ ; $(d)C_f\sqrt{\mathrm{Re}_x}$

Where

$\theta$ ：momentum thickness

$\delta^*$ ：displacement thickness

$\delta$ ：boundary - layer thickness

$C_f$ ：friction coefficient

$\mathrm{Re}_x$ ：Reynolds number ( $\frac{\rho U x}{\mu}$ )

$U$ ：uniform freestream velocity

(5) 20%

Consider the control volume of a thin water layer on an inclined roof ($15^\circ$) during a raining day whose rain speed is $5[m/s]$, the mass flow rate per unit roof area is $\dot{m}/A_{roof} = 0.1[kg/(s\cdot m^2)]$; $\rho = 1000[kg/m^3]$, $g = 9.81[m/s^2]$

1. Write the mass conservation and momentum balance for the control volume shown in the schematic diagram (8%).
2. Estimate $dh/dx = ?$ (5%)
3. Assume a parabolic velocity distribution, say

   $$u = a(x)[hy - y^2/2]$$

   Find the governing equation of the coefficient $a(x)$ if $a(0) = 0$, neglect the effect of the surface tension, $\mu = 1.01\times10^{-3}[N\cdot s/m^2]$, and $\nu = 1.01\times10^{-6}[m^2/s]$.(7%)

$V_{vertical} = -5[m/s]$, $\dot{m}/A_{roof} = 0.1[kg/(s\cdot m^2)]$