1. Given a unit vector $\mathbf{u}$ which is rotating with the angular velocity Ω. Explain why the changing rate of $\mathbf{u}$ is

$$\frac{d\mathbf{u}}{dt} = \Omega \times \mathbf{u}$$

(20%)

2. As shown in Fig. p2, a particle of mass m is moving in a uniform gravitational field with gravity acceleration $-g\mathbf{j}$ and is subject to a linear damping force $-c\mathbf{v}$, where $\mathbf{v}$ is the velocity and c is a constant. If the initial states are $x(0) = x_0$, $y(0) = y_0$, $v_x(0) = v_{x_0}$, and $v_y(0) = v_{y_0}$, determine $x(t)$, $y(t)$, $v_x(t)$, and $v_y(t)$, where t is the time. (20%)

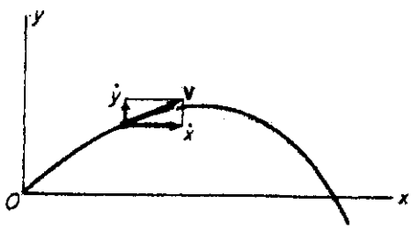


Figure p2: Schematic diagram for Problem 2.

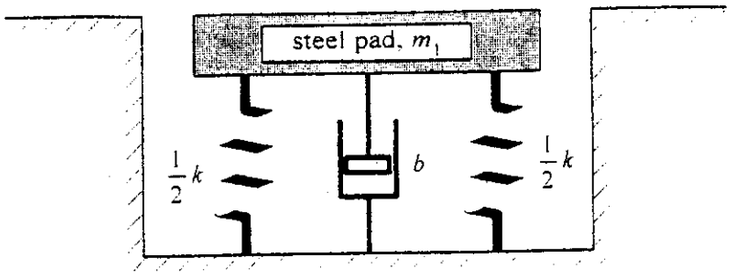
3. The equations of motion for a rigid body can be found in the form of Euler's Equations under some assumptions. Describe these assumptions in deriving Euler's Equations. (10%)

4. A simple spinning top is under the influence of the gravitational torque. Assume that the pivot point of the top is fixed on a table; describe the motion of the top. (10%)

（背面仍有題目，請繼續作答）

5. An isolated test pad may be represented by the model sketched in the figure. This is a steel pad mounted on relatively soft springs, and designed to have an undamped natural frequency of 0.5 Hz. *Note that both the two springs supporting the pad have a spring constant of* $0.5k$. *In addition, the light damping of the springs is represented in the model by the dashpot.*

(a) Find the static deflection of the steel pad under the mass of the pad. (10%)
(b) Write down the equation of motion of the steel pad. (10%)



6. From a height of 1 meter, a steel ball of mass m_2 is dropped into the center of the steel pad in Problem (5), and then caught by a person on the first bounce. Find the resulting motion of the steel pad. *Assume that the impact of the ball on the pad is perfectly elastic and occurs during a negligibly short time. We will also consider only vertical motion of the pad. In addition, let* $m_2 = 0.01\, m_1$ *and the damping ratio of the test pad equals* 0.05. (20%)